

# ARMA-GARCH modelling and white noise tests

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## Abstract

This vignette illustrates applications of white noise tests in GARCH modelling. It is based on an example from an MMath project by the first author.

*Keywords:* autocorrelations, white noise tests, IID tests, GARCH models, time series.

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## 1. The data

In this example we consider data from Freddie Mac, a mortgage loan company in the USA. This stock is an interesting case for study. In the financial crash of 2008 it dropped from roughly \$60 to \$0.5 over the course of a year. It is now (April 2017) majority owned by the government and has all its profits and dividends swept. There has been speculation on this stock being returned to private ownership for years making it prone to clusters of volatility. We import weekly data from Yahoo Finance covering the period from 10/05/2006 to 22/04/2017, and calculate the weekly simple log returns.

```
R> ## using a saved object, originally imported with:  
R> ## FMCC <- yahooSeries("FMCC", from = "2006-05-10", to = "2017-04-22",  
R> ##                               freq = "weekly")  
R> FMCC <- readRDS(system.file("extdata", "FMCC.rds", package = "sarima"))  
R> logreturns <- diff(log(FMCC$FMCC.Close))
```

A plot of the log-returns. is given in Fig. 1. We also calculate the autocorrelations and partial autocorrelations for the log returns.

```
R> FMCClr.acf <- autocorrelations(logreturns)  
R> FMCClr.pacf <- partialAutocorrelations(logreturns)
```

## 2. Autocorrelations

We now produce a plot of the autocorrelations to assess whether the series is autocorrelated, see Fig. 2. There are two bounds plotted on the graph. The straight red line represents the standard bounds under the strong white noise assumption. The second line is under the hypothesis that the process is GARCH.

Several autocorrelations seem significant under the iid hypothesis. This may lead us to fitting an ARMA or ARMA-GARCH model. On the other hand, the autocorrelations are well into

```
R> plot(logreturns, type="l", main="Log-returns of FMCC")
```

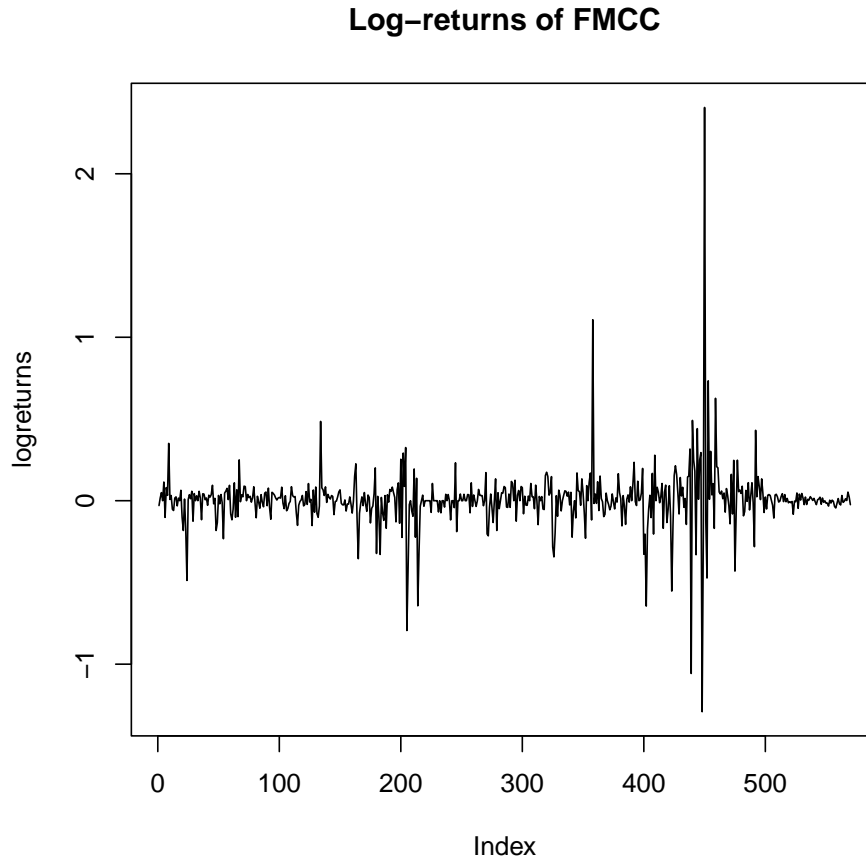


Figure 1: Log-returns of weekly log-returns of FMCC from 10 May 2006 to 22 Apr 2017.

the bands produced under the GARCH hypothesis, suggesting a pure GARCH model, without any ARMA terms. So, it matters on which test we base our decision.

The partial autocorrelation function can be used instead of the autocorrelations, with similar inferences, see Fig. 3.

### 3. Pormanteau tests

Routine portmanteau tests, such as Ljung-Box, also reject the IID hypothesis. Here we carry out IID tests using the method of Li-McLeod:

```
R> wntLM <- whiteNoiseTest(FMCC1r.acf, h0 = "iid", nlags = c(5,10,20),
+                           x = logreturns, method = "LiMcLeod")
R> wntLM$test
```

	ChiSq	DF	pvalue
[1,]	37.18469	5	5.499929e-07

```
R> plot(FMCClr.acf, data = logreturns)
```

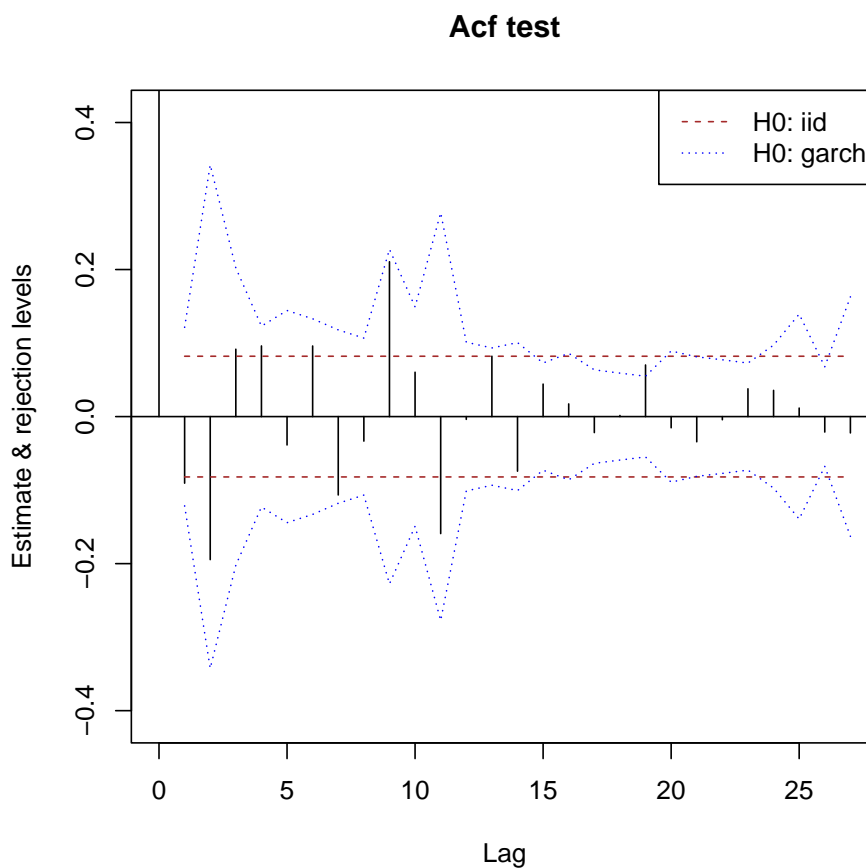


Figure 2: Autocorrelation test of the log returns of FMCC

```
[2,] 76.99131 10 1.946524e-12
[3,] 103.19392 20 3.363466e-13
attr("method")
[1] "LiMcLeod"
```

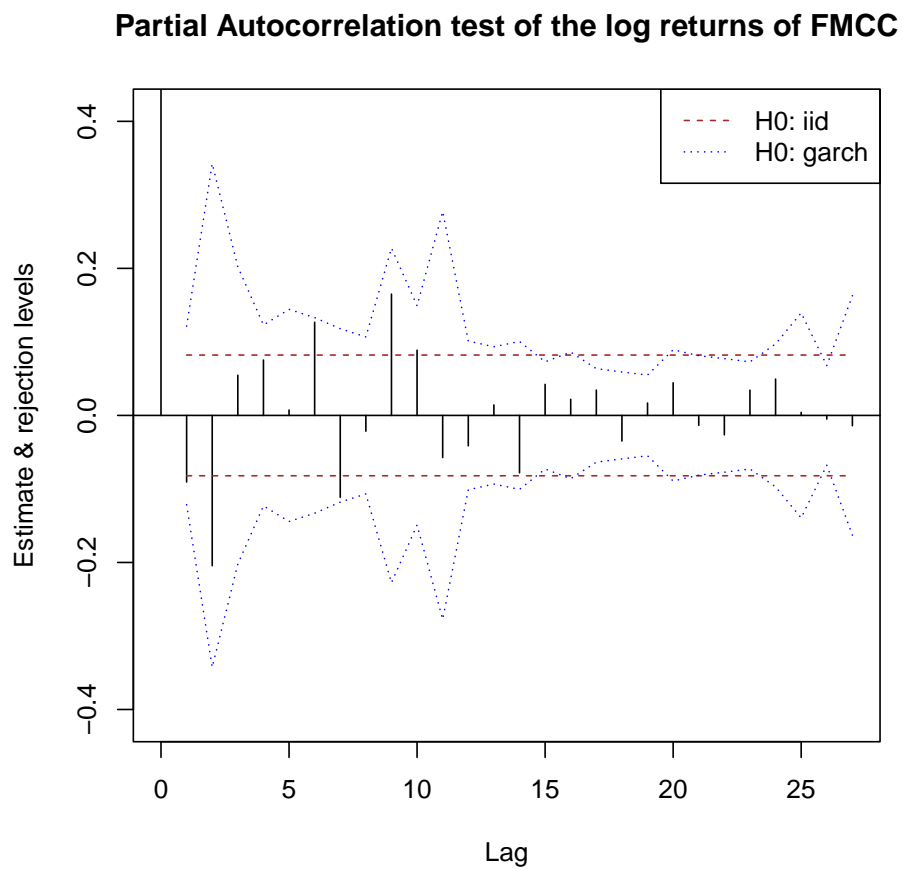
Small p-values lead to rejection of the null hypothesis at reasonable levels. Rejection of the null hypothesis is often taken to mean that the data are autocorrelated.

Let us test for fitting a GARCH-type model by using the following code which has the weaker assumption that the log returns are GARCH. Let us change the null hypothesis to "garch" (one possible weak white noise hypothesis):

```
R> wntg <- whiteNoiseTest(FMCClr.acf, h0 = "garch", nlags = c(5,10,15), x = logreturns)
R> wntg$test
```

```
      h      Q      pval
[1,]  5  4.338367 0.5017961
```

```
R> plot(FMCClr.pacf, data = logreturns,  
+ main="Partial Autocorrelation test of the log returns of FMCC")
```



```
[2,] 10 10.318035 0.4130480
[3,] 15 16.522535 0.3481985
```

The high p-values give no reason to reject the hypothesis that the log-returns are a GARCH white noise process. In other words, there is no need to ARMA modelling.

#### 4. Fitting GARCH(1,1) models and their variants

Based on the discussion above, we go on to fit GARCH model(s), starting with a GARCH(1,1) model with Gaussian innovations.

```
R> fit1 <- garchFit(~garch(1,1), data = logreturns, trace = FALSE)
R> summary(fit1)
```

```
Title:
  GARCH Modelling
```

```
Call:
  garchFit(formula = ~garch(1, 1), data = logreturns, trace = FALSE)
```

```
Mean and Variance Equation:
  data ~ garch(1, 1)
<environment: 0x00000194b6e08c38>
 [data = logreturns]
```

```
Conditional Distribution:
  norm
```

```
Coefficient(s):
      mu      omega    alpha1    beta1
0.006865 0.001658 1.000000 0.328690
```

```
Std. Errors:
  based on Hessian
```

```
Error Analysis:
      Estimate Std. Error t value Pr(>|t|)
mu      0.0068650  0.0031504   2.179  0.02933 *
omega   0.0016580  0.0005068   3.271  0.00107 **
alpha1  1.0000000  0.1452152   6.886 5.72e-12 ***
beta1   0.3286902  0.0797419   4.122 3.76e-05 ***
---
```

```
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
Log Likelihood:
 450.174    normalized: 0.789779
```

Description:

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Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi <sup>2</sup>	900.8757243	0.00000000
Shapiro-Wilk Test	R	W	0.9106544	0.00000000
Ljung-Box Test	R	Q(10)	13.2560009	0.20970867
Ljung-Box Test	R	Q(15)	22.1434228	0.10409801
Ljung-Box Test	R	Q(20)	33.0510414	0.03330812
Ljung-Box Test	R <sup>2</sup>	Q(10)	5.6287621	0.84542949
Ljung-Box Test	R <sup>2</sup>	Q(15)	5.9991295	0.97976237
Ljung-Box Test	R <sup>2</sup>	Q(20)	10.0036245	0.96810617
LM Arch Test	R	TR <sup>2</sup>	5.2750614	0.94815499

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
-1.565523	-1.535027	-1.565621	-1.553624

The diagnostics suggest that the standardised residuals and their squares are IID and that the ARCH effects have been accommodated by the model. Their distribution is clearly not Gaussian however (see the p-values for Jarque-Bera and Shapiro-Wilk Tests), so another conditional distribution can be tried.

Another possible problem is that  $\alpha_1 + \beta_1 > 0$ .

```
R> fit2 <- garchFit(~garch(1,1), cond.dist = c("sstd"), data = logreturns, trace = FALSE)
R> summary(fit2)
```

Title:

GARCH Modelling

Call:

```
garchFit(formula = ~garch(1, 1), data = logreturns, cond.dist = c("sstd"),
  trace = FALSE)
```

Mean and Variance Equation:

```
data ~ garch(1, 1)
```

```
<environment: 0x00000194bf556028>
```

```
[data = logreturns]
```

Conditional Distribution:

```
sstd
```

Coefficient(s):

mu	omega	alpha1	beta1	skew	shape
0.0018471	0.0026688	1.0000000	0.4620441	0.9079461	2.4756746

Std. Errors:  
based on Hessian

Error Analysis:

	Estimate	Std. Error	t value	Pr(> t )
mu	0.001847	0.003229	0.572	0.56726
omega	0.002669	0.001177	2.268	0.02335 *
alpha1	1.000000	0.348326	2.871	0.00409 **
beta1	0.462044	0.099023	4.666	3.07e-06 ***
skew	0.907946	0.041135	22.072	< 2e-16 ***
shape	2.475675	0.228177	10.850	< 2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:

533.9942 normalized: 0.9368319

Description:

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Standardised Residuals Tests:

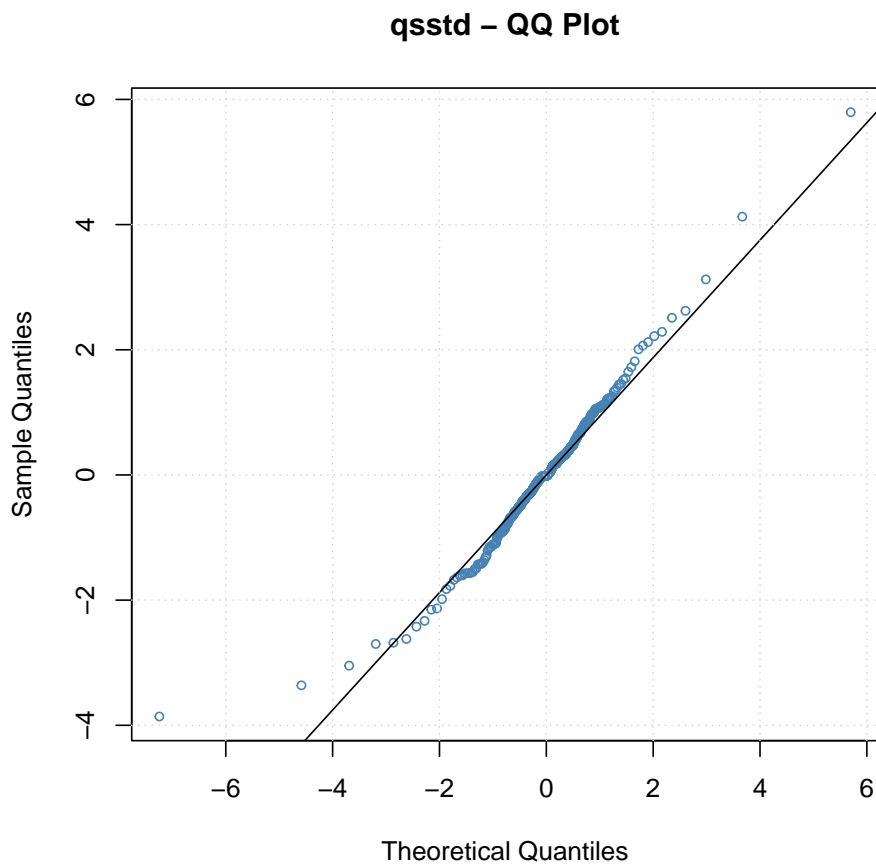
			Statistic	p-Value
Jarque-Bera Test	R	Chi <sup>2</sup>	1470.4746319	0.00000000
Shapiro-Wilk Test	R	W	0.8957822	0.00000000
Ljung-Box Test	R	Q(10)	13.3160246	0.20653531
Ljung-Box Test	R	Q(15)	22.2680019	0.10096683
Ljung-Box Test	R	Q(20)	32.9703035	0.03399477
Ljung-Box Test	R <sup>2</sup>	Q(10)	4.1662019	0.93953797
Ljung-Box Test	R <sup>2</sup>	Q(15)	4.4351483	0.99592473
Ljung-Box Test	R <sup>2</sup>	Q(20)	7.8854508	0.99259886
LM Arch Test	R	TR <sup>2</sup>	3.9797691	0.98379869

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
-1.852611	-1.806868	-1.852830	-1.834764

The qq-plot of the standardised residuals, suggests that the fitted standardised skew-t conditional distribution is not good enough.

R> plot(fit2, which = 13)



```
R> fit3 <- garchFit(~aparch(1,1), cond.dist = c("sstd"), data = logreturns, trace = FALSE)
R> summary(fit3)
```

Title:

GARCH Modelling

Call:

```
garchFit(formula = ~aparch(1, 1), data = logreturns, cond.dist = c("sstd"),
  trace = FALSE)
```

Mean and Variance Equation:

```
data ~ aparch(1, 1)
```

```
<environment: 0x00000194bff652e8>
```

```
[data = logreturns]
```

Conditional Distribution:

```
sstd
```

Coefficient(s):

```
mu      omega      alpha1      gamma1      beta1      delta
```



```

0.0041452  0.0362511  0.2904195  -0.0990915  0.7840805  0.3541800
      skew      shape
0.9265297  2.0896805

```

Std. Errors:  
based on Hessian

Error Analysis:

	Estimate	Std. Error	t value	Pr(> t )
mu	0.0041452	0.0004618	8.976	<2e-16 ***
omega	0.0362511	0.0191449	1.894	0.0583 .
alpha1	0.2904195	0.1490884	1.948	0.0514 .
gamma1	-0.0990915	0.2039870	-0.486	0.6271
beta1	0.7840805	0.0647231	12.114	<2e-16 ***
delta	0.3541800	0.2896686	1.223	0.2214
skew	0.9265297	0.0303459	30.532	<2e-16 ***
shape	2.0896805	0.1320765	15.822	<2e-16 ***

---

Signif. codes: 0 '\*\*\*' 0.001 '\*\*' 0.01 '\*' 0.05 '.' 0.1 ' ' 1

Log Likelihood:

527.6944      normalized: 0.9257797

Description:

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Standardised Residuals Tests:

			Statistic	p-Value
Jarque-Bera Test	R	Chi <sup>2</sup>	6.612007e+06	0.0000000
Shapiro-Wilk Test	R	W	9.472643e-02	0.0000000
Ljung-Box Test	R	Q(10)	3.446815e+00	0.9688735
Ljung-Box Test	R	Q(15)	3.480243e+00	0.9990042
Ljung-Box Test	R	Q(20)	3.697079e+00	0.9999758
Ljung-Box Test	R <sup>2</sup>	Q(10)	5.862782e-03	1.0000000
Ljung-Box Test	R <sup>2</sup>	Q(15)	5.883596e-03	1.0000000
Ljung-Box Test	R <sup>2</sup>	Q(20)	5.906275e-03	1.0000000
LM Arch Test	R	TR <sup>2</sup>	6.639170e+00	0.8805091

Information Criterion Statistics:

AIC	BIC	SIC	HQIC
-1.823489	-1.762498	-1.823876	-1.799692

The qq-plots of the standardised results for all models fitted above suggest that the chosen conditional distributions are unsatisfactory. Moreover, the fitted standardised-t distributions have shape parameters (degrees of freedom) slightly over two. Suggesting extremely heavy tails, maybe even the need for stable distributions.

Note also that in all models above  $\alpha_1 + \beta_1$  is greater than one, a possible violation of any form of stationarity.

Or maybe, it is just that the GARCH models tried here are not able to accomodate varying behaviour before, during and after the financial crisis.

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