

Package ‘freealg’

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Type Package

Title The Free Algebra

Version 1.1-0

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Depends R (>= 3.5.0), methods

Description The free algebra in R with non-commuting indeterminates.

Uses 'disordR' discipline

(Hankin, 2022, <[doi:10.48550/ARXIV.2210.03856](https://doi.org/10.48550/ARXIV.2210.03856)>). To cite the package in publications please use Hankin (2022)

<[doi:10.48550/ARXIV.2211.04002](https://doi.org/10.48550/ARXIV.2211.04002)>.

License GPL (>= 2)

LazyData yes

Imports Rcpp (>= 1.0-7), partitions (>= 1.9-22), mathjaxr, disordR (>= 0.0-9-3)

LinkingTo Rcpp

SystemRequirements C++11

Suggests knitr, testthat, magrittr, markdown, rmarkdown

VignetteBuilder knitr

URL <https://github.com/RobinHankin/freealg>

BugReports <https://github.com/RobinHankin/freealg/issues>

RdMacros mathjaxr

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freealg-package	<i>The Free Algebra</i>
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Description

The free algebra in R with non-commuting indeterminates. Uses 'disordR' discipline (Hankin, 2022, <doi:10.48550/ARXIV.2210.03856>). To cite the package in publications please use Hankin (2022) <doi:10.48550/ARXIV.2211.04002>.

Details

The DESCRIPTION file:

```

Package:          freealg
Type:             Package
Title:            The Free Algebra
Version:          1.1-0
Authors@R:       person(given=c("Robin", "K. S."), family="Hankin", role = c("aut","cre"), email="hankin.robin@
Maintainer:       Robin K. S. Hankin <hankin.robin@gmail.com>
Depends:          R (>= 3.5.0), methods
Description:      The free algebra in R with non-commuting indeterminates. Uses 'disordR' discipline (Hankin, 20
License:          GPL (>= 2)
LazyData:        yes
Imports:          Rcpp (>= 1.0-7), partitions (>= 1.9-22), mathjaxr, disordR (>= 0.0-9-3)
LinkingTo:        Rcpp
SystemRequirements: C++11
Suggests:         knitr,testthat,magrittr,markdown,rmarkdown
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URL:              https://github.com/RobinHankin/freealg
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RdMacros:         mathjaxr
Author:           Robin K. S. Hankin [aut, cre] (<https://orcid.org/0000-0001-5982-0415>)

```

Index of help topics:

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subs	Substitution
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Author(s)

NA

Maintainer: Robin K. S. Hankin <hankin.robin@gmail.com>

Examples

```

a <- as.freealg("x+xyx")
b <- as.freealg("4x +XyX") # upper-case interpreted as inverse

a+b
stopifnot(a+b==b+a) # should be TRUE

a*b ==b*a # FALSE; noncommutative algebra

as.freealg("1+X+xy")^3

rfalg()
rfalg()^2

```

 abelianize

Abelianize a freealg object

Description

Function `abelianize()` returns a `freealg` object that is equivalent to its argument under assumption of Abelianness. The symbols are placed in alphabetical order.

Usage

```
abelianize(x)
```

Arguments

`x` A `freealg` object

Details

Abelianizing a free group element means that the symbols can commute past one another. Abelianization is vectorized.

Value

Returns an object of class `freealg`.

Note

There is a very similar function in the **freegroup** package.

Author(s)

Robin K. S. Hankin

Examples

```
abelianize(as.freealg("ba + 2abba + 3abAB"))
abelianize(.[rfa1g(),rfa1g()])
```

accessor

Accessor methods for freealg objects

Description

Accessor methods for free algebra objects

Usage

```
words(x)
coeffs(x)
coeffs(x) <- value
```

Arguments

<code>x</code>	Object of class <code>freealg</code>
<code>value</code>	Numeric vector of length 1

Details

Access or set the different parts of an `freealg` object. The constant term is technically a coefficient but is documented under `constant.Rd`.

Note

There is an extended discussion of this issue in the `mvp` object at `accessor.Rd`.

Author(s)

Robin K. S. Hankin

See Also[constant](#)**Examples**

```

a <- rfalg()
a
coeffs(a)
words(a) # Note hash is identical to that of coeffs(a)

coeffs(a) <- 7 # replacement methods work
a
coeffs(a) #

```

adjoint*The adjoint map*

Description

The adjoint ad_X of X is a map from a Lie group G to the endomorphism group of G defined by

$$\text{ad}_X(Y) = [X, Y]$$

Usage

```
ad(x)
```

Arguments

`x` Object nominally of class `freealg` but other classes accepted where they make sense

Details

details here

Note

Vignette `adjoint` gives more description

Author(s)

Robin K. S. Hankin

Examples

```
x <- rfalg()
y <- rfalg()

f <- ad(x)
f(y)

f(f(y)) # [x,[x,y]]
```

constant

The constant term

Description

Get and set the constant term of a freealg object

Usage

```
## S3 method for class 'freealg'
constant(x)
## S3 method for class 'numeric'
constant(x)
## S3 replacement method for class 'freealg'
constant(x) <- value
is.constant(x)
```

Arguments

x	Object of class freealg
value	Scalar value for the constant

Details

The constant term in a free algebra object is the coefficient of the empty term. In a freealg object, the map including $\{\} \rightarrow v$ implies that v is the constant.

If x is a freealg object, `constant(x)` returns the value of the constant in the multivariate polynomial; if x is numeric, it returns a constant freealg object with value x .

Function `is.constant()` returns TRUE if its argument has no variables and FALSE otherwise.

Setting the coefficients of the empty freealg returns the zero (empty) object.

Author(s)

Robin K. S. Hankin

Examples

```
p <- as.freealg("1+X+Y+xy")

constant(p)
constant(p^5)

constant(p) <- 1000
p
```

 deriv

Differentiation of freealg objects

Description

Differentiation of freealg objects

Usage

```
## S3 method for class 'freealg'
deriv(expr, r, ...)
```

Arguments

expr	Object of class freealg
r	Integer vector. Elements denote variables to differentiate with respect to. If r is a character vector, it is interpreted as a=1, b=2, . . . , z=26; if of length 1, "aab" is interpreted as c("a", "a", "b")
...	Further arguments, currently ignored

Details

Experimental function $\text{deriv}(S, v)$ returns $\frac{\partial^r S}{\partial v_1 \partial v_2 \dots \partial v_r}$. The Leibniz product rule

$$(u \cdot v)' = uv' + u'v$$

operates even if (as here) u, v do not commute. For example, if we wish to differentiate $aaba$ with respect to a , we would write $f(a) = aaba$ and then

$$f(a + \delta a) = (a + \delta a)(a + \delta a)b(a + \delta a)$$

and working to first order we have

$$f(a + \delta a) - f(a) = (\delta a)aba + a(\delta a)ba + aab(\delta a).$$

In the package:

```
> deriv(as.freealg("aaba"), "a")
free algebra element algebraically equal to
+ 1*aab(da) + 1*a(da)ba + 1*(da)aba
```

A term of a freealg object can include negative values which correspond to negative powers of variables. Thus:

```
> deriv(as.freealg("AAAA"), "a")
free algebra element algebraically equal to
- 1*AAAA(da)A - 1*AAA(da)AA - 1*AA(da)AAA - 1*A(da)AAAA
```

(see also the examples). Vector r may include negative integers which mean to differentiate with respect to the inverse of the variable:

```
> deriv(as.freealg("3abcbCC"), "C")
free algebra element algebraically equal to
+ 3*abcbC(dC) + 3*abcb(dC)C - 3*abc(dC)cbCC
```

It is possible to perform repeated differentiation by passing a suitable value of r. For $\frac{\partial^2}{\partial a \partial c}$:

```
> deriv(as.freealg("aaabAcx"), "ac")
free algebra element algebraically equal to
- 1*aaabA(da)A(dc)x + 1*aa(da)bA(dc)x + 1*a(da)abA(dc)x + 1*(da)aabA(dc)x
```

The infinitesimal indeterminates (“da” etc) are represented by SHRT_MAX+r, where r is the integer for the symbol, and SHRT_MAX is the maximum short integer. This includes negative r. So the maximum number for any symbol is SHRT_MAX. Inverse elements such as A, being represented by negative integers, have differentials that are SHRT_MAX-r.

Function deriv() calls helper function lowlevel_diffn() which is documented at Ops.freealg.Rd.

A vignette illustrating this concept and furnishing numerical verification of the code in the context of matrix algebra is given at inst/freealg_matrix.Rmd.

Author(s)

Robin K. S. Hankin

Examples

```
deriv(as.freealg("4*aaaabaacAc"), 1)

x <- rfalg()
deriv(x, 1:3)

y <- rfalg(7, 7, 17, TRUE)

deriv(y, 1:5) - deriv(y, sample(1:5)) # should be zero
```


dot-class

Class "dot"

Description

The dot object is defined so that idiom like `.[x,y]` returns the commutator, that is, $xy-yx$ or the Lie bracket $[x,y]$. It would have been nice to use $[x,y]$ (that is, without the dot) but although this is syntactically consistent, it cannot be done in R.

The "meat" of the package is:

```
setClass("dot", slots = c(ignore='numeric'))
`.` <- new("dot")
setMethod("[", signature(x="dot", i="ANY", j="ANY"), function(x,i,j,drop){i*j-j*i})
```

The package code includes other bits and pieces such as informative error messages for idiom such as `.[]`. The package defines a matrix method for the dot object. This is because `*` returns (incorrectly, in my view) the elementwise product, not the matrix product.

The Jacobi identity, satisfied by any associative algebra, is

$$[x, [y, z]] + [y, [z, x]] + [z, [x, y]] = 0$$

and the left hand side is returned by `jacobi()`, which should be zero (for some definition of "zero").

Function `ad()` returns the adjoint operator. The adjoint vignette provides details and examples of the adjoint operator.

The dot object is generated by running script `inst/dot.Rmd`, which includes some further discussion and technical documentation, and creates file `dot.rda` which resides in the `data/` directory.

Value

Always returns an object of the same class as `xy`.

Slots

`ignore`: Object of class "numeric", just a formal placeholder

Methods

```
[ signature(x = "dot", i = "ANY", j = "ANY"): ...
[ signature(x = "dot", i = "ANY", j = "missing"): ...
[ signature(x = "dot", i = "function", j = "function"): ...
[ signature(x = "dot", i = "matrix", j = "matrix"): ...
[ signature(x = "dot", i = "missing", j = "ANY"): ...
[ signature(x = "dot", i = "missing", j = "missing"): ...
```

Author(s)

Robin K. S. Hankin

Examples

```

.[as.freealg("x"),as.freealg("y")]
.[as.freealg("x"),as.freealg("y+2z")]
.[as.freealg("x+y+2xYx"),as.freealg("x+y+2xYx")]

x <- rfalg()
y <- rfalg()
z <- rfalg()

jacobi(x,y,z) # Jacobi identity
.[x,.[y,z]] + .[y,.[z,x]] + .[z,.[x,y]] # Jacobi, expanded

f <- ad(x)
f(y)

rM <- function(...){matrix(sample(1:9,9),3,3)} # a random matrix

M <- rM()
N <- rM()
O <- rM()

.[M,N]
jacobi(M,N,O)

```

drop

Drop redundant information

Description

Coerce constant free algebra objects to numeric

Usage

```
drop(x)
```

Arguments

x Free algebra object

Details

If its argument is a constant freealg object, coerce to numeric.

Note

A few functions in the package take drop as an argument which, if TRUE, means that the function returns a dropped value.

Author(s)

Robin K. S. Hankin

See Also[constant](#), [coeffs](#)**Examples**

```
drop(linear(1:5))
drop(4+linear(1:5)*0)
```

 freealg

The free algebra

Description

Create, test for, and coerce to, freealg objects

Usage

```
freealg(words, coeffs)
is_ok_free(words, coeffs)
is.freealg(x)
as.freealg(x, ...)
char_to_freealg(ch)
natural_char_to_freealg(string)
string_to_freealg(string)
vector_to_free(v, coeffs)
```

Arguments

words	Terms of the algebra object, eg $c(1, 2, -1, -3, -2)$ corresponds to $abACB$ because $a = 1, b = 2$ etc; uppercase, or negative number, means inverse
coeffs	Numeric vector corresponding to the coefficients of each element of the word list
string	Character string
ch	Character vector
v	Vector of integers
x	Object possibly of class freealg
...	Further arguments, passed to the methods

Details

Function `freealg()` is the formal creation mechanism for freealg objects. However, it is not very user-friendly; it is better to use `as.freealg()` in day-to-day use.

Function `is_ok_freealg()` checks for consistency of its arguments.

A freealg object is a two-element list. The first element is a list of integer vectors representing the indices and the second is a numeric vector of coefficients. Thus, for example:

```
> as.freealg("a+4bd+3abbbbc")
free algebra element algebraically equal to
+ 1*a + 3*abbbbc + 4*bd
> dput(as.freealg("a+4bd+3abbbbc"))
structure(list(indices = list(1L, c(1L, 2L, 2L, 2L, 2L, 3L),
  c(2L, 4L)), coeffs = c(1, 3, 4)), class = "freealg")
```

Observe that the order of the terms is not preserved and indeed is undefined (implementation-specific). Zero entries are stripped out.

Character strings may be coerced to freealg objects; `as.freealg()` calls `natural_char_to_freealg()`, which is user-friendly. Functions `char_to_freealg()` and `string_to_freealg()` are low-level helper functions. These functions assume that upper-case letters are the multiplicative inverses of the lower-case equivalents; so for example `as.freealg("aA")` and `as.freealg(aBcCbA)` evaluate to one. This can be confusing with the default print method.

Even though individual symbols have multiplicative inverses, a general element of the free algebra will not have a multiplicative inverse. For example, $1+x$ does not have an inverse. The free algebra is not a division algebra, in general.

Note

Internally, the package uses signed integers and as such can have `.Machine$integer.max` different symbols; on my machine this is 2147483647. Of course the print method cannot deal with this as it only has 26 symbols for letters a-z (and A-Z for the inverses), but the objects themselves do not care about the print method. Note also that the experimental calculus facility (as per `deriv()`) reserves numbers in the range $\text{SHRT_MAX} \pm r$ for infinitesimals, where r is the integer for a symbol. This system might change in the future.

Author(s)

Robin K. S. Hankin

Examples

```
freealg(list(1:2, 2:1, numeric(0), 1:6), 1:4)

freealg(sapply(1:5, seq_len), 1:5)

freealg(replicate(5, sample(-5:5, rgeom(1, 1/5), replace=TRUE)), 1:5)

as.freealg("1+xaX")^5
```

Description

The free algebra \mathcal{B} is a *graded* algebra: that is, for each integer $n \geq 0$ there is a homogeneous subspace \mathcal{B}_n with $\mathcal{B}_0 = \mathcal{R}$ and

$$\mathcal{B} = \bigoplus_{n=0}^{\infty} \mathcal{B}_n, \quad \text{and} \quad \mathcal{B}_n \mathcal{B}_m \subseteq \mathcal{B}_{n+m} \quad \text{for all } m, n \geq 0.$$

The elements of $\cup_{n \geq 0} \mathcal{B}_n$ are called *homogeneous* and those of \mathcal{B}_n are called homogenous of degree (or grade) n .

The grade of a term is the number of symbols in it. Thus the grade of xxx and $4xy$ is 3; the grade of a constant is zero. Because the terms are stored in an implementation-specific way, the grade of a multi-term object is a disorder object.

The grade of the zero freealg object, `grade(as.freealg(0))`, is defined to be zero, which ensures that `max(grades(abelianize(x))) <= max(grades(x))` is always satisfied. However, a case for NULL could be made.

Usage

```
grades(x)
grade(x,n)
grade(x,n) <- value
```

Arguments

x	Freealg object
n	Integer vector
value	Replacement value, a numeric vector

Details

`grades(x)` returns the grade (number of symbols) in each term of a freealg object x.

`grade(x,n)` returns the freealg object comprising terms with grade n (which may be a vector). Note that this function is considerably less efficient than `clifford::grade()`.

`grade(x,n) <- value` sets the coefficients of terms with grade n. For value, a length-one numeric vector is accepted (notably zero, which kills terms of grade n) and also a freealg object comprising terms of grade coden.

Value

Returns a disorder object

Note

A similar concept *grade* is discussed in the **clifford** package

Author(s)

Robin K. S. Hankin

References

H. Munthe-Kaas and B. Owren 1999. “Computations in a free Lie algebra”, *Phil. Trans. R. Soc. Lond. A*, 357:957–981 (theorem 3.8)

Examples

```
X <- as.freealg("1 -x + 5*y + 6*x*y -8*x*x*x*x*y*x")
X
grades(X)

a <- rfalg(30)
a
grades(a)
grade(a,2)
grade(a,2) <- 0 # kill all grade-2 terms
a

grade(a,1) <- grade(a,1) * 888
a
```

horner

Horner's method

Description

Horner's method for multivariate polynomials

Usage

```
horner(P, v)
```

Arguments

P	Free algebra polynomial
v	Numeric vector of coefficients

Details

This function is (almost) the same as `mvp : horner()`.

Given a polynomial

$$p(x) = a_0 + a_1 + a_2x^2 + \cdots + a_nx^n$$

it is possible to express $p(x)$ in the algebraically equivalent form

$$p(x) = a_0 + x (a_1 + x (a_2 + \cdots + x (a_{n-1} + xa_n) \cdots))$$

which is much more efficient for evaluation, as it requires only n multiplications and n additions, and this is optimal. Function `horner()` will take a `freealg` object for its first argument.

Author(s)

Robin K. S. Hankin

Examples

```

horner("x", 1:4) # note constant term is 1.

horner("x+y",1:3) # note presence of xy and yx terms

horner("1+x+xyX",1:3)

```

 linear

A simple free algebra object

Description

Create simple free algebra objects including linear expressions, for example

```

> linear(1:3)
free algebra element algebraically equal to
+ 1*a + 2*b + 3*c
> linear(1:3,power=5)
free algebra element algebraically equal to
+ 1*aaaaa + 2*bbbbb + 3*ccccc
>

```

Usage

```
linear(x,power=1)
```

Arguments

x	Numeric vector of terms
power	Integer vector of powers

Note

Many of the functions documented at `mvp::special.Rd` do not make sense in the context of the free algebra. Function `mvp::product()`, for example, imposes an order on the expansion.

Function `constant()` is documented at `constant.Rd`, but is listed below for convenience.

Author(s)

Robin K. S. Hankin

See Also

[constant](#), [zero](#)

Examples

```
linear(1:3)
linear(1:3,power=5)
linear(1:3,power=3:1)
```

nterms	<i>Number of terms in a freealg object</i>
--------	--

Description

Number of terms in a freealg object; number of coefficients

Usage

```
nterms(x)
```

Arguments

x	Freealg object
---	----------------

Value

Returns a non-negative integer

Author(s)

Robin K. S. Hankin

Examples

```
(a <- freealg(list(1:3,4:7,1:10),1:3))
nterms(a)
nterms(a+1)
nterms(a*0)
```

Ops.freealg	<i>Arithmetic Ops methods for the the free algebra</i>
-------------	--

Description

Arithmetic operators for manipulation of freealg objects such as addition, multiplication, powers, etc

Usage

```
## S3 method for class 'freealg'
Ops(e1, e2)
free_negative(S)
free_power_scalar(S,n)
free_eq_free(e1,e2)
free_plus_numeric(S,x)
free_plus_free(e1,e2)
lowlevel_simplify(words,coeffs)
lowlevel_free_prod(words1,coeffs1,words2,coeffs2)
lowlevel_free_sum(words1,coeffs1,words2,coeffs2)
lowlevel_free_power(words,coeffs,n)
lowlevel_diffn(words,coeffs,r)
lowlevel_subs(words1, coeffs1, words2, coeffs2, r)
inv(S)
```

Arguments

S, e1, e2	Objects of class freealg
n	Integer, possibly non-positive
r	Integer vector indicating variables to differentiate with respect to
x	Scalar value
words, words1, words2	A list of words, that is, a list of integer vectors representing the variables in each term
coeffs, coeffs1, coeffs2	Numeric vector representing the coefficients of each word

Details

The function `Ops.freealg()` passes binary arithmetic operators (“+”, “-”, “*”, “^”, and “==”) to the appropriate specialist function.

The caret, as in a^n , denotes arithmetic exponentiation, as in $x^3 == x*x*x$. The only comparison operators are equality and inequality; $x == y$ is defined as `is.zero(x-y)`.

Functions `lowlevel_foo()` are low-level functions that interface directly with the C routines in the `src/` directory and are not intended for the end-user.

Function `inv()` is defined only for freealg objects with a single term. If x has a single term we have $inv(x)*x = x*inv(x) = 1$. There is no corresponding division in the package because a/b may be either $a*inv(b)$ or $inv(b)*a$.

Author(s)

Robin K. S. Hankin

Examples

```
rfalg()
as.freealg("1+x+xy+yx") # variables are non-commutative
as.freealg("x") * as.freealg("X") # upper-case letters are lower-case inverses

constant(as.freealg("x+y+X+Y")^6) # OEIS sequence A035610
```

```
inv(as.freealg("2aabAAAAx"))
```

```
pepper
```

Combine variables in every possible order

Description

Given a list of variables, construct every term comprising only those variables; function `pepper()` returns a free algebra object equal to the sum of these terms.

The function is named for a query from an exam question set by Sarah Marshall in which she asked how many ways there are to arrange the letters of word “pepper”, the answer being $\binom{6}{1\ 2\ 3} = \frac{6!}{1!2!3!} = 60$.

Function `multiset()` in the `partitions` package gives related functionality.

Usage

```
pepper(v)
```

Arguments

`v` Variables to combine. If a character string, coerce to variable numbers

Author(s)

Robin K. S. Hankin

See Also

[linear](#)

Examples

```
pepper(c(1,2,2,2,3))
pepper("pepper")
```

```
print
```

Print freealg objects

Description

Print methods for free algebra objects

Usage

```
## S3 method for class 'freealg'
print(x,...)
```

Arguments

x Object of class freealg in the print method
 ... Further arguments, currently ignored

Note

The print method does not change the internal representation of a freealg object, which is a two-element list, the first of which is a list of integer vectors representing words, and the second is a numeric vector of coefficients.

The print method has special dispensation for length-zero freealg objects but these are not handled entirely consistently.

The print method is sensitive to the value of `getOption("usecaret")`, defaulting to "no". The default is to use uppercase letters to represent multiplicative inverses, but if TRUE, inverses are indicated using "⁻¹". This becomes cumbersome for powers above the first. For example, the default notation for aba^{-2} is abAA but becomes $aba^{-1}a^{-1}$ if usecaret is TRUE.

Integers exceeding SHRT_MAX are reserved for infinitesimals, which are printed as "da"; see the note at `deriv.Rd` for details.

Author(s)

Robin K. S. Hankin

See Also

[freealg,deriv](#)

Examples

```
rfalg()

x <- rfalg(inc=TRUE)
x                    # default
options("usecaret" = TRUE) # use caret
x
options("usecaret" = FALSE) # back to the default
x
```

 rfalg

Random free algebra objects

Description

Random elements of the free algebra, intended as quick "get you going" examples of freealg objects

Usage

```
rfalg(n=7, distinct=3, maxsize=4, include.negative=FALSE)
```

Arguments

<code>n</code>	Number of terms to generate
<code>distinct</code>	Number of distinct symbols to use
<code>maxsize</code>	Maximum number of symbols in any word
<code>include.negative</code>	Boolean, with default FALSE meaning to use only positive symbols (lower-case letters) and TRUE meaning to use upper-case letters as well, corresponding to the inverse of the lower-case symbols

Details

What you see is what you get, basically. A term such as aaBaAbaC will be simplified to aaaC.

Author(s)

Robin K. S. Hankin

Examples

```
rfalg()
rfalg(include.negative=TRUE)^2
```

```
constant(rfalg())
```

subs

Substitution

Description

Substitute symbols in a freealg object for numbers or other freealg objects

Usage

```
subs(S, ...)
subsu(S1, S2, r)
```

Arguments

<code>S, S1, S2</code>	Objects of class freealg
<code>r</code>	Integer specifying symbol to substitute ($a = 1, b = 2$ etc)
<code>...</code>	named arguments corresponding to variables to substitute

Details

Function `subs()` substitutes variables for `freealg` objects (coerced if necessary) using a natural R idiom. Observe that this type of substitution is sensitive to order:

```
> subs("ax", a="1+x", x="1+a")
free algebra element algebraically equal to
+ 2 + 3*a + 1*aa
```

```
> subs("ax", x="1+a", a="1+x")
free algebra element algebraically equal to
+ 2 + 3*x + 1*xx
```

Functions `subsu()` is a lower-level formal function, not really intended for the end-user. Function `subsu()` takes `S1` and substitutes occurrences of symbol `r` with `S2`.

No equivalent to `mvp : subvec()` is currently implemented.

Value

Returns a `freealg` object.

Note

Function `subs()` is one place in the package where the use of letters is effectively hard-wired in. Idiom such as

```
subs("abccc", b="1+3x")
```

is very nice, but identifies “b” with 2. Note that argument `r` of `subsu()` is canonically an integer but a single character is interpreted as a letter. See also the note at `freealg.Rd`.

Author(s)

Robin K. S. Hankin

Examples

```
subs("abccc", b="1+3x")
subs("aaaa", a="1+x") # binomial
```

```
subs("abA", b=31)
```

```
subs("1+a", a="A") # can substitute for an inverse
subs("A", a="1+x") # inverses are not substituted for
```

```
## Sequential substitution works:
```

```
subs("abccc", b="1+3x", x="1+d+2e")
subs(rfalg(), a=rfalg())
```

zero

The zero algebraic object

Description

Test for a freealg object's being zero

Usage

```
is.zero(x)
```

Arguments

x Object of class freealg

Details

Function `is.zero()` returns TRUE if x is indeed the zero free algebra object. It is defined as `length(coeffs(x))==0` for reasons of efficiency, but conceptually it returns `x==constant(0)`.

(Use `constant(0)` to create the zero object).

Author(s)

Robin K. S. Hankin

See Also

[constant](#)

Examples

```
stopifnot(is.zero(constant(0)))
```

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