

# Package ‘SequenceSpikeSlab’

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**Type** Package

**Title** Exact Bayesian Model Selection Methods for the Sparse Normal Sequence Model

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**Author** Steven de Rooij [aut],  
Tim van Erven [cre, aut],  
Botond Szabo [aut]

**Maintainer** Tim van Erven <tim@timvanerven.nl>

**Description** Contains fast functions to calculate the exact Bayes posterior for the Sparse Normal Sequence Model, implementing the algorithms described in Van Erven and Szabo (2021, <[doi:10.1214/20-BA1227](https://doi.org/10.1214/20-BA1227)>). For general hierarchical priors, sample sizes up to 10,000 are feasible within half an hour on a standard laptop. For beta-binomial spike-and-slab priors, a faster algorithm is provided, which can handle sample sizes of 100,000 in half an hour. In the implementation, special care has been taken to assure numerical stability of the methods even for such large sample sizes.

**License** GPL (>= 2)

**Imports** Rcpp (>= 0.12.18), RcppProgress (>= 0.4.1), selectiveInference (>= 1.2.5)

**LinkingTo** Rcpp, RcppProgress

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fast\_spike\_slab\_beta    *Compute marginal posterior estimates for beta-spike-and-slab prior*

---

### Description

Computes marginal posterior probabilities (slab probabilities) that data points have non-zero mean for the spike-and-slab prior with a Beta(beta\_kappa,beta\_lambda) prior on the mixing parameter. The posterior mean is also provided.

### Usage

```
fast_spike_slab_beta(
  x,
  sigma = 1,
  m = 20,
  slab = "Laplace",
  Laplace_lambda = 0.5,
  Cauchy_gamma = 1,
  beta_kappa = 1,
  beta_lambda,
  show_progress = TRUE
)
```

### Arguments

x	Vector of n data points
sigma	Standard deviation of the Gaussian noise in the data. May also be set to "auto", in which case sigma is estimated using the estimateSigma function from the selectiveInference package

m	The number of discretization points used is proportional to $m\sqrt{n}$ . The larger m, the better the approximation, but the runtime also increases linearly with m. The default $m=20$ usually gives sufficient numerical precision.
slab	Slab distribution. Must be either "Laplace" or "Cauchy".
Laplace_lambda	Parameter of the Laplace slab
Cauchy_gamma	Parameter of the Cauchy slab
beta_kappa	Parameter of the beta-distribution
beta_lambda	Parameter of the beta-distribution. Default value= $n+1$
show_progress	Boolean that indicates whether to show a progress bar

### Details

The run-time is  $O(m\sqrt{n}^3)$  on  $n$  data points, which means that doubling the size of the data leads to an increase in computation time by approximately a factor of  $2\sqrt{2}=2.8$ . Data sets of size  $n=100,000$  should be feasible within approximately 30 minutes.

### Value

list (postprobs, postmean, sigma), where postprobs is a vector of marginal posterior slab probabilities that  $x[i]$  has non-zero mean for  $i = 1, \dots, n$ ; postmean is a vector with the posterior mean for the  $x[i]$ ; and sigma is the value of sigma (this may be of interest when the sigma="auto" option is used)

### Examples

```
# Illustrate that fast_spike_slab_beta is a faster way to compute the same results as
# general_sequence_model on the beta-binomial prior

# Generate data
n <- 500          # sample size
n_signal <- 25    # number of non-zero theta
A <- 5           # signal strength
theta <- c(rep(A,n_signal), rep(0,n-n_signal))
x <- theta + rnorm(n, sd=1)

# Choose slab
slab <- "Cauchy"
Cauchy_gamma <- 1

cat("Running fast_spike_slab_beta (fast for very large n)...\n")
res_fss <- fast_spike_slab_beta(x, sigma=1, slab=slab, Cauchy_gamma=Cauchy_gamma)

cat("Running general_sequence_model (slower for very large n)...\n")
res_gsm <- general_sequence_model(x, sigma=1, slab=slab,
                                log_prior="beta-binomial", Cauchy_gamma=Cauchy_gamma)

cat("Maximum difference in marginal posterior slab probabilities:",
    max(abs(res_gsm$postprobs - res_fss$postprobs)))
cat("\nMaximum difference in posterior means:",
```

```

max(abs(res_gsm$postmean - res_fss$postmean)), "\n")

# Plot means
M=max(abs(x))+1
plot(1:n, x, pch=20, ylim=c(-M,M), col='green', xlab="", ylab="",
     main="Posterior Means (Same for Both Methods)")
points(1:n, theta, pch=20, col='blue')
points(1:n, res_gsm$postmean, pch=20, col='black', cex=0.6)
points(1:n, res_fss$postmean, pch=20, col='magenta', cex=0.6)
legend("topright", legend=c("general_sequence_model", "fast_spike_slab_beta",
                           "data", "truth"),
      col=c("black", "magenta", "green", "blue"), pch=20, cex=0.7)

```

---

```
general_sequence_model
```

*Compute marginal posterior estimates*

---

## Description

This function computes marginal posterior probabilities (slab probabilities) that data points have non-zero mean for the general hierarchical prior in the sparse normal sequence model. The posterior mean is also provided.

## Usage

```

general_sequence_model(
  x,
  sigma = 1,
  slab = "Laplace",
  log_prior = "beta-binomial",
  Laplace_lambda = 0.5,
  Cauchy_gamma = 1,
  beta_kappa = 1,
  beta_lambda,
  show_progress = TRUE
)

```

## Arguments

x	Vector of n data points
sigma	Standard deviation of the Gaussian noise in the data. May also be set to "auto", in which case sigma is estimated using the estimateSigma function from the selectiveInference package
slab	Slab distribution. Must be either "Laplace" or "Cauchy".
log_prior	Vector of length n+1 containing the logarithms of the prior probabilities $\pi_n(s)$ that the number of spikes is equal to s for $s=0,\dots,n$ . It is allowed to use an unnormalized prior that does not sum to 1, because adding any constant to the log-prior

probabilities does not change the result. Instead of a vector, `log_prior` may also be set to "beta-binomial" as a short-hand for `log_prior = lbeta(beta_kappa+(0:n),beta_lambda+n-(0:n)) - lbeta(beta_kappa,beta_lambda) + lchoose(n,0:n)`.

<code>Laplace_lambda</code>	Parameter of the Laplace slab
<code>Cauchy_gamma</code>	Parameter of the Cauchy slab
<code>beta_kappa</code>	Parameter of the beta-distribution in the beta-binomial prior
<code>beta_lambda</code>	Parameter of the beta-distribution in the beta-binomial prior. Default value= $n+1$
<code>show_progress</code>	Boolean that indicates whether to show a progress bar

## Details

The run-time is  $O(n^2)$  on  $n$  data points, which means that doubling the size of the data leads to an increase in computation time by approximately a factor of 4. Data sets of size  $n=25,000$  should be feasible within approximately 30 minutes.

## Value

list (postprobs, postmean, sigma), where `postprobs` is a vector of marginal posterior slab probabilities that  $x[i]$  has non-zero mean for  $i = 1, \dots, n$ ; `postmean` is a vector with the posterior mean for the  $x[i]$ ; and `sigma` is the value of `sigma` (this may be of interest when the `sigma="auto"` option is used)

## Examples

```
# Experiments similar to those of Castelo, Van der Vaart, 2012

# Generate data
n <- 500          # sample size
n_signal <- 25    # number of non-zero theta
A <- 5           # signal strength
theta <- c(rep(A,n_signal), rep(0,n-n_signal))
x <- theta + rnorm(n, sd=1)

# Choose slab
slab <- "Laplace"
Laplace_lambda <- 0.5

# Prior 1
kappa1 <- 0.4    # hyperparameter
logprior1 <- c(0,-kappa1*(1:n)*log(n*3/(1:n)))
res1 <- general_sequence_model(x, sigma=1,
                              slab=slab,
                              log_prior=logprior1,
                              Laplace_lambda=Laplace_lambda)
print("Prior 1: Elements with marginal posterior probability >= 0.5:")
print(which(res1$postprobs >= 0.5))

# Prior 2
kappa2 <- 0.8    # hyperparameter
logprior2 <- kappa2*lchoose(2*n-0:n,n)
```

```

res2 <- general_sequence_model(x, sigma=1,
                             slab=slab,
                             log_prior=logprior2,
                             Laplace_lambda=Laplace_lambda)
print("Prior 2: Elements with marginal posterior probability >= 0.5:")
print(which(res2$postprobs >= 0.5))

# Prior 3
beta_kappa <- 1 # hyperparameter
beta_lambda <- n+1 # hyperparameter
res3 <- general_sequence_model(x, sigma=1,
                              slab=slab,
                              log_prior="beta-binomial",
                              Laplace_lambda=Laplace_lambda)
print("Prior 3: Elements with marginal posterior probability >= 0.5:")
print(which(res3$postprobs >= 0.5))

# Plot means for all priors
M=max(abs(x))+1
plot(1:n, x, pch=20, ylim=c(-M,M), col='green', xlab="", ylab="", main="Posterior Means")
points(1:n, theta, pch=20, col='blue')
points(1:n, res1$postmean, pch=20, col='black', cex=0.6)
points(1:n, res2$postmean, pch=20, col='magenta', cex=0.6)
points(1:n, res3$postmean, pch=20, col='red', cex=0.6)
legend("topright", legend=c("posterior mean 1", "posterior mean 2", "posterior mean 3",
                           "data", "truth"),
      col=c("black", "magenta", "red", "green", "blue"), pch=20, cex=0.7)

```

---

### SSS\_discrete\_spike\_slab

*Compute marginal posterior probabilities (slab probabilities) that data points have non-zero mean for the discretized spike-and-slab prior.*

---

#### Description

Compute marginal posterior probabilities (slab probabilities) that data points have non-zero mean for the discretized spike-and-slab prior.

#### Usage

```
SSS_discrete_spike_slab(log_phi_psi, dLambda, show_progress = TRUE)
```

#### Arguments

`log_phi_psi` List `{logphi, logpsi}` containing two vectors of the same length `n` that represent a preprocessed version of the data. `logphi` and `logpsi` should contain the logs of the `phi` and `psi` densities of the data points, as produced for instance by [SSS\\_log\\_phi\\_psi\\_Laplace](#) or [SSS\\_log\\_phi\\_psi\\_Cauchy](#)

dLambda	Discretized Lambda prior, as generated by either discretize_Lambda or discretize_Lambda_beta.
show_progress	Boolean that indicates whether to show a progress bar

**Value**

Returns a vector with marginal posterior slab probabilities that  $x[i]$  has non-zero mean for  $i = 1, \dots, n$ .

---

SSS\_discretize\_Lambda *Given a prior Lambda on the alpha-parameter in the spike-and-slab model, make a discretized version of Lambda that is only supported on a grid of approximately  $m * \sqrt{n}$  discrete values of alpha. This discretized version of Lambda is required as input for SSS\_discrete\_spike\_slab. NB Lambda needs to satisfy a technical condition from the paper that guarantees its density does not vary too rapidly. For Lambda=Beta(kappa,lambda) use SSS\_discretize\_Lambda\_beta instead.*

---

**Description**

Given a prior Lambda on the alpha-parameter in the spike-and-slab model, make a discretized version of Lambda that is only supported on a grid of approximately  $m * \sqrt{n}$  discrete values of alpha. This discretized version of Lambda is required as input for SSS\_discrete\_spike\_slab. NB Lambda needs to satisfy a technical condition from the paper that guarantees its density does not vary too rapidly. For Lambda=Beta(kappa,lambda) use SSS\_discretize\_Lambda\_beta instead.

**Usage**

```
SSS_discretize_Lambda(m = 20, n, log_Lambda_cdf)
```

**Arguments**

m	A multiplier for the number of discretization points
n	The sample size
log_Lambda_cdf	A function that takes as input a value of alpha and calculates the log of the cumulative distribution function of Lambda at alpha

**Value**

List (alpha\_grid, log\_probs), where alpha\_grid is a vector with the generated grid points, and log\_probs are the logs of the prior probabilities of these grid points for the discretized Lambda prior.

---

SSS\_discretize\_Lambda\_beta

*Given prior  $\Lambda = \text{Beta}(\kappa, \lambda)$  on the alpha-parameter in the spike-and-slab model, make a discretized version of  $\Lambda$  that is only supported on a grid of approximately  $m * \sqrt{n}$  discrete values of alpha. This discretized version of  $\Lambda$  is required as input for SSS\_discrete\_spike\_slab.*

---

### Description

Given prior  $\Lambda = \text{Beta}(\kappa, \lambda)$  on the alpha-parameter in the spike-and-slab model, make a discretized version of  $\Lambda$  that is only supported on a grid of approximately  $m * \sqrt{n}$  discrete values of alpha. This discretized version of  $\Lambda$  is required as input for SSS\_discrete\_spike\_slab.

### Usage

```
SSS_discretize_Lambda_beta(m = 20, n, kappa, lambda)
```

### Arguments

m	A multiplier for the number of discretization points
n	The sample size
kappa	Parameter of the prior. Needs to be at least 0.5.
lambda	Parameter of the prior. Needs to be at least 0.5.

### Value

List (alpha\_grid, log\_probs), where alpha\_grid is a vector with the generated grid points, and log\_probs are the logs of the prior probabilities of these grid points for the discretized  $\Lambda$  prior.

---

SSS\_hierarchical\_prior

*Compute marginal posterior probabilities (slab probabilities) that data points have non-zero mean for the hierarchical prior.*

---

### Description

Compute marginal posterior probabilities (slab probabilities) that data points have non-zero mean for the hierarchical prior.

### Usage

```
SSS_hierarchical_prior(log_phi_psi, logprior, show_progress = TRUE)
```



**Arguments**

log_phi_psi	List {logphi, logpsi} containing two vectors of the same length $n$ that represent a preprocessed version of the data. logphi and logpsi should contain the logs of the phi and psi densities of the data points, as produced for instance by <a href="#">SSS_log_phi_psi_Laplace</a> or <a href="#">SSS_log_phi_psi_Cauchy</a>
logprior	vector of length $n+1$ with components $\text{logprior}[p]=\log(\pi_n(p))$ for $p = 0, \dots, n$
show_progress	Boolean that indicates whether to show a progress bar

**Value**

Returns a vector with marginal posterior slab probabilities that  $x[i]$  has non-zero mean for  $i = 1, \dots, n$ .

---

**SSS\_hierarchical\_prior\_binomial**

*Compute marginal posterior probabilities (slab probabilities) that data points have non-zero mean using the general hierarchical prior algorithm, but specialized to the Beta[kappa,lambda]-binomial prior. This function is equivalent to calling [SSS\\_hierarchical\\_prior](#) with  $\text{logprior} = \text{lbeta}(\text{kappa}+(0:n), \text{lambda}+n-(0:n)) - \text{lbeta}(\text{kappa}, \text{lambda}) + \text{lchoose}(n, 0:n)$ , but more convenient when using the Beta[kappa,lambda]-binomial prior and with a minor interior optimization that avoids calculating the choose explicitly.*

---

**Description**

Compute marginal posterior probabilities (slab probabilities) that data points have non-zero mean using the general hierarchical prior algorithm, but specialized to the Beta[kappa,lambda]-binomial prior. This function is equivalent to calling [SSS\\_hierarchical\\_prior](#) with  $\text{logprior} = \text{lbeta}(\text{kappa}+(0:n), \text{lambda}+n-(0:n)) - \text{lbeta}(\text{kappa}, \text{lambda}) + \text{lchoose}(n, 0:n)$ , but more convenient when using the Beta[kappa,lambda]-binomial prior and with a minor interior optimization that avoids calculating the choose explicitly.

**Usage**

```
SSS_hierarchical_prior_binomial(
  log_phi_psi,
  kappa,
  lambda,
  show_progress = TRUE
)
```

**Arguments**

log_phi_psi	List {logphi, logpsi} containing two vectors of the same length $n$ that represent a preprocessed version of the data. logphi and logpsi should contain the logs of the phi and psi densities of the data points, as produced for instance by <a href="#">SSS_log_phi_psi_Laplace</a> or <a href="#">SSS_log_phi_psi_Cauchy</a>
-------------	---

kappa	First parameter of the beta-distribution
lambda	Second parameter of the beta-distribution
show_progress	Boolean that indicates whether to show a progress bar

**Value**

Returns a vector with marginal posterior slab probabilities that  $x[i]$  has non-zero mean for  $i = 1, \dots, n$ .

---

SSS\_log\_phi\_psi\_Cauchy

*Calculate log of phi and psi marginal densities for Cauchy(gamma) slab*

---

**Description**

Calculate log of densities phi and psi for data vector x, where

$$phi[i] = Normal(x[i], sigma^2)$$

$$psi[i] = E_{Cauchy}(\theta)[Normal(x[i] - \theta, sigma^2)]$$

**Usage**

SSS\_log\_phi\_psi\_Cauchy(x, sigma, gamma)

**Arguments**

x	data vector
sigma	standard deviation of observations
gamma	parameter of Cauchy slab density

**Value**

list (phi, psi), containing logs of phi and psi densities

---

SSS\_log\_phi\_psi\_Laplace

*Calculate log of phi and psi marginal densities for Laplace(lambda) slab*

---

**Description**

Calculate log of densities phi and psi for data vector x, where

$$\begin{aligned} \phi[i] &= \text{Normal}(x[i], \sigma^2) \\ \psi[i] &= E_{\text{Laplace}(\theta)}[\text{Normal}(x[i] - \theta, \sigma^2)] \end{aligned}$$

**Usage**

SSS\_log\_phi\_psi\_Laplace(x, sigma, lambda)

**Arguments**

x	data vector
sigma	standard deviation of observations
lambda	parameter of Laplace slab density

**Value**

list (phi, psi), containing logs of phi and psi densities

---

SSS\_make\_beta\_grid *Creates a vector of uniformly spaced grid points in the beta parametrization Ensures the number of generated grid points is >= mingridpoints (which does not have to be integer), and that their number is always odd so there is always a grid point at pi/4.*

---

**Description**

Creates a vector of uniformly spaced grid points in the beta parametrization Ensures the number of generated grid points is >= mingridpoints (which does not have to be integer), and that their number is always odd so there is always a grid point at pi/4.

**Usage**

SSS\_make\_beta\_grid(minngridpoints)

**Arguments**

minngridpoints Minimum number of grid points

**Value**

Vector of betagrid points

---

SSS\_postmean\_Cauchy    *Compute posterior means of data points for the Cauchy(gamma) slab*

---

**Description**

Compute posterior means of data points for the Cauchy(gamma) slab

**Usage**

SSS\_postmean\_Cauchy(x, logpsi, postprobs, sigma, gamma)

**Arguments**

x	Data vector of length n
logpsi	Vector of length n that represents a preprocessed version of the data. It should contain the logs of the psi densities of the data points, as produced by <a href="#">SSS_log_phi_psi_Cauchy</a> .
postprobs	Vector of marginal posterior slab probabilities that $x[i]$ has non-zero mean for $i = 1, \dots, n$ .
sigma	standard deviation of observations
gamma	parameter of Cauchy slab density

**Value**

Vector of n posterior means

---

SSS\_postmean\_Laplace    *Compute posterior means of data points for the Laplace(lambda) slab*

---

**Description**

Compute posterior means of data points for the Laplace(lambda) slab

**Usage**

SSS\_postmean\_Laplace(x, logpsi, postprobs, sigma, lambda)

**Arguments**

x	Data vector of length n
logpsi	Vector of length n that represents a preprocessed version of the data. It should contain the logs of the psi densities of the data points, as produced by <a href="#">SSS_log_phi_psi_Laplace</a> .
postprobs	Vector of marginal posterior slab probabilities that $x[i]$ has non-zero mean for $i = 1, \dots, n$ .
sigma	standard deviation of observations
lambda	parameter of Laplace slab density

**Value**

Vector of n posterior means

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