

Package ‘PEIP’

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Description

Several functions introduced in Aster et al.'s book on inverse theory. The functions are often translations of MATLAB code developed by the authors to illustrate concepts of inverse theory as applied to geophysics. Generalized inversion, tomographic inversion algorithms (conjugate gradients, 'ART' and 'SIRT'), non-linear least squares, first and second order Tikhonov regularization, roughness constraints, and procedures for estimating smoothing parameters are included.

License GPL (>= 2)

NeedsCompilation no

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R topics documented:

PEIP-package	2
Ainv	3
art	4
bartl	5
bayes	6
blf2	7
cgl	9
chi	10
chi2cdf	11
chi2inv	12
dcost	13

error.bar	13
flipGSVD	15
gcv	16
gcv_function	17
get_l_rough	18
ginv	19
GSVD	20
idcost	22
imagesc	23
interp2grid	24
irls	25
irlsl1reg	26
kac	28
linesconst	29
lmarq	30
loadMAT	31
l_curve_corner	32
l_curve_tgsvd	33
l_curve_tikh_gsvd	35
l_curve_tikh_svd	37
mcmc	38
Mnorm	41
nnz	42
occam	42
phi	43
phiinv	44
picard_vals	45
plotconst	46
quadlin	47
rnk	48
setDesignG	49
shawG	50
sirt	51
tin	52
USV	53
Vnorm	54
vspprofile	55
Index	57

Description

Auxilliary functions and routines for running the examples and excersizes described in the book on inverse theory.

Details

Package: PEIP
 Type: Package
 Version: 1.3
 Date: 2012-07-30
 License: GPL

These functions are used in conjunction with the example described in the PEIP book.

There is one C-code routine, interp2grid. This is introduced to replicate the MATLAB code interp2. It does not work exactly as the matlab code prescribes.

In the PEIP library one LAPACK routine is called: dggsvd. In R, LAPACK routines are stored in slightly different locations on Linux, Windows and Mac computers. Be aware. This will come up in examples from Chapter 4.

Almost all examples work as scripts run with virtually no user input, e.g.

Author(s)

Jonathan M. Lees<jonathan.lees.edu> Maintainer:Jonathan M. Lees<jonathan.lees.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Ainv

An Inverse Solution

Description

QR decomposition solution to $Ax=b$

Usage

Ainv(GAB, x, tol = 1e-12)

Arguments

GAB	design matrix
x	right hand side
tol	tolerance for singularity

Details

I needed something to make up for the lame-o matlab code that does this $h = G \setminus x$ to get the inverse

Value

Inverse Solution

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
set.seed(2015)
GAB = matrix(runif(36), ncol=6)
truex = rnorm(ncol(GAB))
rhs = GAB %*% truex

rhs = as.vector(rhs )

tout = Ainv(GAB, rhs, tol = 1e-12)
tout - truex
```

art

ART Inverse solution

Description

ART algorithm for solving sparse linear inverse problems

Usage

```
art(A, b, tol, maxiter)
```

Arguments

A	Constraint matrix
b	right hand side
tol	difference tolerance for successive iterations (stopping criteria)
maxiter	maximum iterations (stopping criteria).

Details

Alpha is a damping factor. If $\alpha < 1$, then we won't take full steps in the ART direction. Using a smaller value of alpha (say $\alpha = .75$) can help with convergence on some problems.

Value

x solution

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
set.seed(2015)
G = setDesignG()
### % Setup the true model.
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

### % reshape the true model to be a vector
mtruev=as.vector(mtruem);

### % Compute the data.
dtrue=G %*% mtruev;

### % Add the noise.

d=dtrue+0.01*rnorm(length(dtrue));

mkac<-art(G,d,0.01,200)
par(mfrow=c(1,2))
imagesc(matrix(mtruem,16,16) , asp=1 , main="True Model" );

imagesc(matrix(mkac,16,16) , asp=1 , main="ART Solution" );
```

bartl

Bartlett window

Description

Bartlett (triangle) window of length m

Usage

```
bartl(m)
```

Arguments

m integer, length of vector

Value

vector

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
bartl(11)
```

bayes	<i>Bayes Inversion</i>
-------	------------------------

Description

Given a linear inverse problem $Gm=d$, a prior mean $mprior$ and covariance matrix $covm$, data d , and data covariance matrix $covd$, this function computes the MAP solution and the corresponding covariance matrix.

Usage

```
bayes(G, mprior, covm, d, covd)
```

Arguments

G Design Matrix
mprior vector, prior model
covm vector, model covariance
d vector, right hand side
covd vector, data covariance

Value

vector model

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
## Not run:
set.seed(2015)
G = setDesignG()
###
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

###
mtruev=as.vector(mtruem);
imagesc(matrix(mtruem,16,16) , asp=1 , main="True Model" );

matrix(mtruem,16,16) , asp=1 , main="True Model" )

###
dtrue=G %*% mtruev;

###
d=dtrue+0.01*rnorm(length(dtrue));
covd = 0.1*diag( nrow=length(d) )
covm = 1*diag( nrow=dim(G)[2] )

## End(Not run)
```

Description

Bounded least squares

Usage

```
blf2(A, b, c, delta, l, u)
```

Arguments

A	Design Matrix
b	Right hand side
c	matrix weight on x
delta	tolerance
l	lower bound
u	upper bound

Details

Solves the problem: $\min/\max c'x$ where $\|Ax-b\| \leq \delta$ and $l \leq x \leq u$.

Value

x	solution
---	----------

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Stark, P.B. , and R. L. Parker, *Bounded-Variable Least-Squares: An Algorithm and Applications*, Computational Statistics 10:129-141, 1995.

Examples

```
### set up an inverse problem:Shaw problem

n = 20
G = shawG(n,n)

spike = rep(0,n)
spike[10] = 1

spiken = G %*% spike

wts = rep(1, n)
```



```
delta = 1e-03
set.seed(2015)
dspiken = spiken + 6e-6 * rnorm(length(spiken))

lb = spike - (.2) * wts
ub = spike + (.2) * wts

dspiken = dspiken

blf2(G, dspiken, wts , delta, lb, ub)
```

cglS

Conjugate gradient Least squares

Description

Conjugate gradient Least squares

Usage

```
cglS(Gmat, dee, niter)
```

Arguments

Gmat	input matrix
dee	right hand side
niter	max number of iterations

Details

Performs niter iterations of the CGLS algorithm on the least squares problem $\min \text{norm}(G^*m-d)$. Gmat should be a sparse matrix.

Value

X	matrix of models
rho	misfit norms
eta	model norms

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
set.seed(11)
#### perfect data with no noise
n <- 5
A <- matrix(runif(n*n),nrow=n)
B <- runif(n)
### get right-hand-side (data)
trhs = as.vector( A %*% B )
Lout = cglS(A, trhs , 15)

### solution is
Lout$X[,15]

Lout$X[,15] - B
```

chi

Chi function

Description

Chi function

Usage

chi(x, n)

Arguments

x	value
n	degrees of freedom

Value

function evaluated

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
x = seq(0, 10, length=100)
n = 5
y=chi(x, n)
plot(x, y)
```

`chi2cdf`*Chi-Sq CDF*

Description

Computes the Chi² CDF, using a transformation to N(0,1) on page 333 of Thisted, Elements of Statistical Computing.

Usage

```
chi2cdf(x, n)
```

Arguments

x	end value of chi ² pdf to integrate to. (scalar)
n	degrees of freedom (scalar)

Details

Note that x and m must be scalars.

Value

p	probability that Chi ² random variable is less than or equal to x (scalar).
---	--

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
x= seq(from=0.1, to=0.9, length=20)
chi2cdf(x , 3)
```

`chi2inv`*Inverse Chi-Sq*

Description

Inverse Chi-Sq

Usage`chi2inv(x, n)`**Arguments**

`x` probability that Chi² random variable is less than or equal to `x` (scalar).
`n` degrees of freedom (scalar)

Details

Computes the inverse Chi² distribution corresponding to a given probability that a Chi² random variable with the given degrees of freedom is less than or equal to `x`. Uses `chi2cdf.m`.

Value

corresponding value of `x` for given probability.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

See Also

`chi`, `chi2cdf`

Examples

```
x = seq(from=0.1, to=0.9, length=10)
h = chi2cdf(x, 3)

chi2inv(h, 3)
```

dcost	<i>cosine transform</i>
-------	-------------------------

Description

Computes the column-by-column discrete cosine transform of X.

Usage

```
dcost(X)
```

Arguments

X Time series matrix

Value

cosine transformed data

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
x <- 1:4  
  
### compare fft with cosine transform  
fft(x)  
  
dcost(x)
```

error.bar	<i>Plot Error Bar</i>
-----------	-----------------------

Description

Plot Error Bar

Usage

```
error.bar(x, y, lo, hi, pch = 1, col = 1, barw = 0.1, add = FALSE, ...)
```

Arguments

x	X-values
y	Y-values
lo	Lower limit of error bars
hi	Upper limit of error bars
pch	plotting character
col	color
barw	width of the bar
add	logical, add=FALSE starts a new plot
...	other plotting parameters

Value

graphical side effects

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
x = 1:10
y = 2*x+5
zup = rnorm(10)
```

```
zup = zup-min(zup)+.5
zdown = rnorm(10)
zdown = zdown-min(zdown)+.2
```

```
#### example with same error on either side:
error.bar(x, y, y-zup, y+zup, pch = 1, col = 'brown' , barw = 0.1, add =
FALSE)
```

```
#### example with different error on either side:
error.bar(x, y, y-zdown, y+zup, pch = 1, col = 'brown' , barw = 0.1, add
= FALSE)
```

`flipGSVD`*Flip output of GSVD*

Description

Flip (reverse order) output of GSVD

Usage

```
flipGSVD(vs, d1 = c(50, 50), d2 = c(48, 50))
```

Arguments

<code>vs</code>	list output of GSVD
<code>d1</code>	dimensionals of A
<code>d2</code>	dimensions of B

Details

This flipping of the matrix is done to agree with the Matlab code.

Value

Same as GSVD, but order of eigenvectors is reversed.

<code>U</code>	m by m orthogonal matrix
<code>V</code>	p by p orthogonal matrix, $p=\text{rank}(B)$
<code>X</code>	n by n nonsingular matrix
<code>C</code>	singular values, m by n matrix with diagonal elements shifted from main diagonal
<code>S</code>	singular values, p by n diagonal matrix

Note

The GSVD routines are from LAPACK.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

GSVD

Examples

```

set.seed(12)

n <- 5
A <- matrix(runif(n*n),nrow=n)
B <- matrix(runif(n*n),nrow=n)

VS = GSVD(A, B)

FVS = flipGSVD(VS, d1 = dim(A) , d2 = dim(B) )
## see that order of eigen vectors is reversed
diag(VS$S)
diag(FVS$S)

```

gcval

Get c-val

Description

Extract the smallest regularization parameter.

Usage

```
gcval(U, s, b, npoints)
```

Arguments

U	U matrix from gsvd(G, L)
s	[diag(C) diag(S)] which are the lambdas and mus from the gsvd
b	the data to try and match
npoints	number of alphas to estimate

Details

Evaluate the GCV function `gcv_function` at `npoints` points.

Value

List:

reg_min	alpha with the minimal g (scalar)
g	$\ Gm_\alpha(L) - d\ ^2 / (\text{Tr}(I - GG^\#))^2$
alpha	alpha for the corresponding g

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

gcv_function

Examples

```
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

lam=sqrt(diag(t(Lam1 %% Lam1)));

mu=sqrt(diag(t(M1)%*%M1));

p=rank(L1);

sm1=cbind(lam[1:p],mu[1:p])

### % get the gcv values varying alpha
###
ngcvpoints=1000;

HI = gcval(U1,sm1,t,ngcvpoints);
```

Description

Auxiliary routine for GCV calculations

Usage

```
gcv_function(alpha, gamma2, beta)
```

Arguments

alpha	parameter
gamma2	square of the gamma from the gsvd
beta	projected data to fit

Value

vector, $g - \|Gm_{\alpha,L} - d\|^2 / (\text{Tr}(I - GG\#)^2)$

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

get_1_rough

One-D Roughening

Description

Returns a 1D differentiating matrix operating on a series with n points.

Usage

```
get_1_rough(n, deg)
```

Arguments

n	integer, number of data points
deg	order of the derivative to approximate

Details

Used to get first and 2nd order roughening matrices for 1-D problems

Value

Matrix:discrete differentiation matrix

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
### first order roughening matrix for a 10 by 10 model: a sparse matrix
N = 10
L1 = get_l_rough(10,1);

### second order roughening matrix for a 10 by 10 model
N = 10
L2 = get_l_rough(10,2);
```

ginv

Get inverse

Description

Get inverse of matrix or solve $Ax=b$.

Usage

```
ginv(G, x, tol = 1e-12)
```

Arguments

G	Design Matrix
x	right hand side
tol	tolerance

Details

This function used as alternative to matlab code that does this $h = G \setminus x$ to get the inverse

Value

inverse as a N by 1 matrix.

Note

Be careful about the usage of tolerance

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

solve, Ainv

Examples

```
set.seed(2015)
GAB = matrix(runif(36), ncol=6)
truex = rnorm(ncol(GAB))
rhs = GAB %*% truex

rhs = as.vector(rhs )

tout = ginv(GAB, rhs, tol = 1e-12)
tout - truex
```

GSVD

Generalized SVD

Description

Wrapper for generalized svd from LAPACK

Usage

GSVD(A, B)

Arguments

A	Matrix, see below
B	Matrix, see below

Details

The A and B matrices will be, $A=U*C*t(X)$ and $B=V*S*t(X)$, respectively.

Since PEIP is based on a book, which is itself based on MATLAB routines, the convention here follows the book. The R implementation uses LAPACK and wraps the function so the output will comply with the book. See page 104 of the second edition of the Aster book cited below. That said, the purpose is to find an inversion of the form $Y = t(A aB)$, where a is a regularization parameter, B is smoothing matrix and A is the design matrix for the forward problem. The input matrices A and B are assumed to have full rank, and $p = \text{rank}(B)$. The generalized singular values are then $\gamma = \lambda/\mu$, where $\lambda = \sqrt{\text{diag}(t(C)*C)}$ and $\mu = \sqrt{\text{diag}(t(S)*S)}$.

Value

U	m by m orthogonal matrix
V	p by p orthogonal matrix, $p=\text{rank}(B)$
X	n by n nonsingular matrix
C	singular values, m by n matrix with diagonal elements shifted from main diagonal
S	singular values, p by n diagonal matrix

Note

Requires R version of LAPACK. The code is a wrapper for the `dggsvd` function in LAPACK. The author thanks Berend Hasselman for advice and help preparing this function.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

See Also

`flipGSVD`

Examples

```
# Example from NAG F08VAF

A <- matrix(1:15, nrow=5, ncol=3)
B <- matrix(c(8,1,6,
              3,5,7,
              4,9,2), nrow=3, byrow=TRUE)

z <- GSVD(A,B)
C <- z$C
S <- z$S
sqrt(diag(t(C) %*% C)) / sqrt(diag(t(S) %*% S))
testA = A - z$U %*% C %*% t(z$X)
testB = B - z$V %*% S %*% t(z$X)

print(testA)
print(testB)
```

`idcost`*Inverse cosine transform*

Description

Takes the column-by-column inverse discrete cosine transform of Y.

Usage

```
idcost(Y)
```

Arguments

Y Input cosine transform

Value

Time series

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

See Also

dcost

Examples

```
x <- 1:4

### compare fft with cosine transform
fft(x)

zig = dcost(x)
zag = idcost(zig)
```

`imagesc`*Image Display*

Description

Display image in matlab format, i.e. flip and transpose.

Usage

```
imagesc(G, col = grey((1:99)/100), ...)  
contoursc(G, ...)
```

Arguments

<code>G</code>	Image matrix
<code>col</code>	color scale
<code>...</code>	graphical parameters

Details

Program flips image and transposes prior to plotting. The contour version does the same and can be used to add contours.

Value

graphical side effects

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);  
  
mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;  
mtruem[10,9]=1; mtruem[10,11]=1;  
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;  
mtruem[2,3]=1; mtruem[2,4]=1;  
mtruem[3,3]=1; mtruem[3,4]=1;  
  
imagesc(mtruem, asp=1)
```

`interp2grid`*Bilinear and Bicubic Interpolation to Grid*

Description

This code includes a bicubic interpolation and a bilinear interpolation adapted from Numerical Recipes in C: The art of scientific computing (chapter 3... bicubic interpolation) and a bicubic interpolation from in java code.

Inputs are a list of points to interpolate to and from raster objects of class 'asc' (adehabitat package), 'RasterLayer' (raster package) or 'SpatialGridDataFrame' (sp package).

Usage

```
interp2grid(mat,xout,yout,xin=NULL,yin=NULL,type=2)
```

Arguments

<code>mat</code>	a matrix of data that can be a raster matrix of class 'asc' (adehabitat package), 'RasterLayer' (raster package) or 'SpatialGridDataFrame' (sp package) NA values are not permitted.. data must be complete.
<code>xout</code>	a vector of data representing x coordinates of the output grid. Resulting grid must have square cell sizes if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
<code>yout</code>	a vector of data representing y coordinates of the output grid. Resulting grid must have square cell sizes if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
<code>xin</code>	a vector identifying the locations of the columns of the input data matrix. These are automatically populated if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
<code>yin</code>	a vector identifying the locations of the rows of the input data matrix. These are automatically populated if mat is of class 'asc', 'RasterLayer' or 'SpatialGridDataFrame'.
<code>type</code>	an integer value representing the type of interpolation method used. 1 - bilinear adapted from Numerical Recipes in C 2 - bicubic adapted from Numerical Recipes in C 3 - bicubic adapted from online java code

Value

Returns a matrix of the originating class.

Author(s)

Jeremy VanDerWal <jjvanderwal@gmail.com>

Examples

```

tx = seq(0,3,0.1)
ty = seq(0,3,0.1)

tmat = matrix(runif(16,1,16),nrow=4)
txin = seq(0,3,length=4)
tyin = seq(0,3,length=4)

bilinear1 = interp2grid(tmat,tx,ty,txin, tyin, type=1)
bicubic2 = interp2grid(tmat,tx,ty,txin, tyin, type=2)
bicubic3 = interp2grid(tmat,tx,ty,txin, tyin, type=3)

par(mfrow=c(2,2),cex=1)
  image(tmat,main='base',zlim=c(0,16),col=heat.colors(100))
  image(bilinear1,main='bilinear',zlim=c(0,16),col=heat.colors(100))
  image(bicubic2,main='bicubic2',zlim=c(0,16),col=heat.colors(100))
  image(bicubic3,main='bicubic3',zlim=c(0,16),col=heat.colors(100))

```

irls*Iteratively reweight least squares*

Description

Uses the iteratively reweight least squares strategy to find an approximate L_p solution to $Ax=b$.

Usage

```
irls(A, b, tolR, tolX, p, maxiter)
```

Arguments

A	Matrix of the system of equations.
b	Right hand side of the system of equations
tolR	Tolerance below which residuals are ignored
tolX	Stopping tolerance. Stop when $(\text{norm}(\text{newx}-x)/(1+\text{norm}(x)) < \text{tolx})$
p	Specifies which p-norm to use (most often, $p=1$.)
maxiter	Limit on number of iterations of IRLS

Details

Use to get L-1 norm solution of inverse problems.

Value

x	Approximate L_p solution
---	----------------------------

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
t = 1:10
y=c(109.3827,187.5385,267.5319,331.8753,386.0535,
428.4271,452.1644,498.1461,512.3499,512.9753)
sigma = rep(8, length(y))
N=length(t);

### % Introduce the outlier
y[4]=y[4]-200;

G = cbind( rep(1, N), t, -1/2*t^2 )

### % Apply the weighting

yw = y/sigma;

Gw = G/sigma

m2 = solve( t(Gw) %*% Gw , t(Gw) %*% yw, tol=1e-12 )

### Solve for the 1-norm solution

m1 = irls(Gw,yw,1.0e-5,1.0e-5,1,25)
m1
```

 irlsl1reg

L1 least squares with sparsity

Description

Solves the system $Gm=d$ using sparsity regularization on Lm . Solves the L1 regularized least squares problem: $\min \text{norm}(G*m-d,2)^2 + \alpha * \text{norm}(L*m,1)$

Usage

```
irlsl1reg(G, d, L, alpha, maxiter = 100, tolx = 1e-04, tolr = 1e-06)
```

Arguments

G	design matrix
d	right hand side
L	regularization matrix
alpha	regularization parameter
maxiter	Maximum number of IRLS iterations
tolx	Tolerance on successive iterates
tolr	Tolerance below which we consider an element of L^*m to be effectively zero

Value

m	model vector
---	--------------

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```

n = 20
G = shawG(n,n)

spike = rep(0,n)
spike[10] = 1

spiken = G %%% spike

wts = rep(1, n)
delta = 1e-03
set.seed(2015)
dspiken = spiken + 6e-6 *rnorm(length(spiken))
L1 = get_l_rough(n,1);
alpha = 0.001

k = irlsl1reg(G, dspiken, L1, alpha, maxiter = 100, tolx = 1e-04, tolr = 1e-06)

plotconst(k,-pi/2,pi/2, ylim=c(-.2, 0.5), xlab="theta", ylab="Intensity" );

```

kac	<i>Kaczmarz</i>
-----	-----------------

Description

Implements Kaczmarz's algorithm to solve a system of equations iteratively

Usage

```
kac(A, b, tolX, maxiter)
```

Arguments

A	Constraint matrix
b	right hand side
tolX	difference tolerance for successive iterations (stopping criteria)
maxiter	maximum iterations (stopping criteria)

Value

x	solution
---	----------

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
set.seed(2015)
G = setDesignG()
### % Setup the true model.
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

### % reshape the true model to be a vector
mtruev=as.vector(mtruem);
```

```
### % Compute the data.
dtrue=G %** mtruev;

### % Add the noise.

d=dtrue+0.1*rnorm(length(dtrue));

mkac<-kac(G,d,0.0,200)
par(mfrow=c(1,2))
imagesc(matrix(mtruev,16,16) , asp=1 , main="True Model" );

imagesc(matrix(mkac,16,16) , asp=1 , main="Kacz Solution" );
```

linesconst

Plot constant model

Description

Add to plotting model in piecewise constant form over n subintervals, where n is the length of x .

Usage

```
linesconst(x, l, r, ...)
```

Arguments

<code>x</code>	model to be plotted
<code>l</code>	left endpoint of plot
<code>r</code>	right endpoint of plot
<code>...</code>	graphical parameters

Details

Used for plotting vector models

Value

graphical side effects

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

plotconst

Examples

```
zip = runif(25)
plotconst(zip, 0, 1 )
linesconst(runif(25) , 0, 1 , col='red' )
```

lmarq

*Lev-Marquardt Inversion***Description**

Use the Levenberg-Marquardt algorithm to minimize $f(p)=\sum(F_i(p)^2)$

Usage

```
lmarq(afun, ajac, p0, tol, maxiter)
```

Arguments

afun	name of the function $F(x)$
ajac	name of the Jacobian function $J(x)$
p0	initial guess
tol	stopping tolerance
maxiter	maximum number of iterations allowed

Value

pstar	best solution found.
iter	Iteration count.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
fun<-function(p){
### Compute the function values.
fvec=rep(0,length(TM))
fvec=(Q*exp(-D^2*p[1]/(4*p[2]*TM))/(4*pi*p[2]*TM) - H)/SIGMA
return(fvec)
}
jac <-function( p)
{
### use known formula for the derivatives in the Jacobian
n=length(TM)
```

```

J= matrix(0,nrow=n,ncol=2)

J[,1]=(-Q*D^2*exp(-D^2*p[1]/(4*p[2]*TM))/(16*pi*p[2]^2*TM^2))/SIGMA

J[,2]=(Q/(4*pi*p[2]^2*TM))*
      ((D^2*p[1]/(4*p[2]*TM)-1)*exp(-D^2*p[1]/(4*p[2]*TM))/SIGMA
return(J)
}

H=c(0.72, 0.49, 0.30, 0.20, 0.16, 0.12)
TM=c(5.0, 10.0, 20.0, 30.0, 40.0, 50.0)

### Fixed parameter values.
D=60
Q=50
### We'll use sigma=1cm.
SIGMA=0.01*rep(1,length(H))
### The unknown/estimated parameters are S=p(1) and T=p(2).
p0=c(0.001, 1.0)
### Solve the least squares problem with LM.
PEST = lmarq('fun','jac',p0,1.0e-12,100)

```

loadMAT

Load a Matlab matfile

Description

Load a Matlab matfile, rename the internal parameters to get R-objects

Usage

```
loadMAT(fn, pos=1)
```

Arguments

fn	file name of MATfile
pos	integer, position in search path, default=1

Details

Program reads in previously saved mat-files and extracts the data, and renames the variables to match the book.

Value

Whatever is in the MATfile

Note

Matfiles are created using the matlab2R routines

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

l_curve_corner

L Curve Corner

Description

Retrieve corner of L-curve

Usage

l_curve_corner(rho, eta, reg_param)

Arguments

rho	misfit
eta	model norm or seminorm
reg_param	regularization parameter

Value

reg_corner	the value of reg_param with maximum curvature
ireg_corner	the index of the value in reg_param with maximum curvature
kappa	the curvature for each reg_param

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```

#### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

K1 = l_curve_tgsvd(U1,t,X1,Lam1,G,L1);

rho1 =K1$rho
eta1 =K1$eta
reg_param1 =K1$reg_param
m1s =K1$m

### % store where the corner is (from visual inspection)
vcorn = l_curve_corner(rho1, eta1, reg_param1)

ireg_corner1=vcorn$reg_corner
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste('1st order reg corner is: ',ireg_corner1));

plot(rho1,eta1,type="b", log="xy" , xlim=c(1e-4, 1e-2) , ylim=c(6e-6, 2e-4) ,
      xlab="Residual Norm ||Gm-d||_2", ylab="Solution Seminorm ||Lm||_2" );
points(rho_corner1, eta_corner1, col='red', cex=2 )

```

l_curve_tgsvd

L curve tgsvd

Description

L curve parameters and models for truncated gsvd regularization.

Usage

```
l_curve_tgsvd(U, d, X, Lam, G, L)
```

Arguments

U	U, output of GSVD
d	output of GSVD
X	output of GSVD
Lam	output of GSVD
G	output of GSVD
L	output of GSVD

Value

List:

eta	the solution seminorm $\ Lm\ $
rho	the residual norm $\ Gm - d\ $
reg_param	corresponding regularization parameters
m	corresponding suite of models for truncated GSVD

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
#### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )
```

```

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

K1 = l_curve_tgsvd(U1,t,X1,Lam1,G,L1);

rho1 =K1$rho
eta1 =K1$eta
reg_param1 =K1$reg_param
m1s =K1$m

### % store where the corner is (from visual inspection)
ireg_corner1=8;
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste('1st order reg corner is: ',ireg_corner1));

plot(rho1,eta1,type="b", log="xy", xlim=c(1e-4, 1e-2) , ylim=c(6e-6, 2e-4) ,
      xlab="Residual Norm ||Gm-d||_2", ylab="Solution Seminorm ||Lm||_2" );

```

l_curve_tikh_gsvd *L-curve tikh gsvd*

Description

L-curve tikh gsvd

Usage

```
l_curve_tikh_gsvd(U, d, X, Lam, Mu, G, L, npoints, varargin = NULL)
```

Arguments

U	from the gsvd
d	data vector for the problem $G*m=d$
X	from the gsvd
Lam	from the gsvd
Mu	from the gsvd

G	system matrix
L	roughening matrix
npoints	
varargin	alpha_min, alpha_max: if specified, constrain the logarithmically spaced regularization parameter range, otherwise an attempt is made to estimate them from the range of generalized singular values

Details

Uses output of GSVD

Value

eta	- the solution seminorm Lm
rho	- the residual norm G m - d
reg_param	- corresponding regularization parameters
m	- corresponding suite of models for truncated GSVD

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
##### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

K1 = l_curve_tikh_gsvd(U1,t,X1,Lam1,M1, G,L1, 25);

rho1 =K1$rho
eta1 =K1$eta
reg_param1 =K1$reg_param
```

```

m1s =K1$m

### store where the corner is (from visual inspection)
ireg_corner1=8;
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste('1st order reg corner is: ',ireg_corner1));

plot(rho1,eta1,type="b", log="xy", xlim=c(1e-4, 1e-2) , ylim=c(6e-6, 2e-4) ,
      xlab="Residual Norm ||Gm-d||_2", ylab="Solution Seminorm ||Lm||_2" );

```

l_curve_tikh_svd *L-curve Tikhonov*

Description

L-curve for Tikhonov regularization

Usage

```
l_curve_tikh_svd(U, s, d, npoints, varargin = NULL)
```

Arguments

U	matrix of data space basis vectors from the svd
s	vector of singular values
d	the data vector
npoints	the number of logarithmically spaced regularization parameters
varargin	alpha_min, alpha_max: if specified, constrain the logarithmically spaced regularization parameter range, otherwise an attempt is made to estimate them from the range of singular values

Details

Calculates the L-curve

Value

eta	the solution norm m or seminorm Lm
rho	the residual norm G m - d
reg_param	corresponding regularization parameters

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
#### Vertical Seismic Profile example
set.seed(2015)
VSP = vspprofile()
t = VSP$t2
G = VSP$G
M = VSP$M
N = VSP$N

L1 = get_l_rough(N,1);
littleU = PEIP::GSVD(as.matrix(G), as.matrix(L1) );

BIGU = flipGSVD(littleU, dim(G), dim(L1) )

U1 = BIGU$U
V1 =BIGU$V
X1=BIGU$X
Lam1=BIGU$C
M1=BIGU$S

K1 = l_curve_tikh_svd(U1, diag(M1) , X1, 25, varargin = NULL)

rho1 =K1$rho
eta1 =K1$eta
reg_param1 =K1$reg_param
m1s =K1$m

### store where the corner is (from visual inspection)
ireg_corner1=8;
rho_corner1=rho1[ireg_corner1];
eta_corner1=eta1[ireg_corner1];
print(paste("1st order reg corner is: ",ireg_corner1));

plot(rho1,eta1,type="b", log="xy" ,
      xlab="Residual Norm ||Gm-d||_2", ylab="Solution Seminorm ||Lm||_2" );
```

mcmc

Maximum likelihood Models

Description

Maximum likelihood Models

Usage

```
mcmc(aologprior, aologlikelihood, agenerate, alogproposal, m0, niter)
```

Arguments

alogprior	Name of a function that computes the log of the prior distribution.
aloglikelihood	Name of a function the computes the log of the likelihood.
agenerate	Name of a function that generates a random model from the current model using the
alogproposal	Name of a function that computes the log of the proposal distribution $r(x,y)$.
m0	Initial model
niter	Number of iterations to perform

Value

mout	MCMC samples
mMAP	Best model found in the MCMC simulation.
accrate	Acceptance rate

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```

fun <-function(m,x)
{
  y=m[1]*exp(m[2]*x)+m[3]*x*exp(m[4]*x)
  return(y)
}

generate <-function( x) {
  y=x+step*rnorm(4)
  return(y)
}

logprior <-function(m)
{
  if( (m[1]>=0) & (m[1]<=2) &
      (m[2]>=-0.9) & (m[2]<=0) &
      (m[3]>=0) & (m[3]<=2) &
      (m[4]>=-0.9) & (m[4]<=0) )
  {
    lp=0
  }
  else
  {
    lp= -Inf
  }

  return(lp)
}
loglikelihood <-function(m)

```

```

{
  fvec=(y-fun(m,x))/sigma
  L=(-1/2)*sum(fvec^2)
  return(L)
}
logproposal <-function(x,y)
{
  LR=(-1/2)*sum((x-y)^2/step^2)
  return(LR)
}

### Generate the data set.
x=seq(from=1, by=0.25, to=7.0)

mtrue=c(1.0, -0.5, 1.0, -0.75)

ytrue=fun(mtrue,x)

sigma=0.01*rep(1, times= length(ytrue) )

y=ytrue+sigma*rnorm(length(ytrue) )

### set the MCMC parameters
### number of skips to reduce autocorrelation of models
skip=100
### burn-in steps
BURNIN=1000
### number of posterior distribution samples
N=4100
### MVN step size
step = 0.005*rep(1,times=4)

### We assume flat priors here
m0 = c(0.9003,
      -0.5377,
      0.2137,
      -0.0280)

alogprior='logprior'
aloglikelihood='loglikelihood'
agenerate='generate'
alogproposal='logproposal'

### ### initialize model at a random point on [-1,1]

### m0=(runif(4)-0.5)*2
### this is the matlab initialization:
m0 = c(0.9003,
      -0.5377,
      0.2137,
      -0.0280)

MM = mcmc('logprior','loglikelihood','generate','logproposal',m0,N)

```



```
mout = MM[[1]]  
mMAP= MM[[2]]  
pacc= MM[[3]]
```

Mnorm

Matrix Norm

Description

Matrix Norm

Usage

```
Mnorm(X, k = 2)
```

Arguments

X	matrix
k	norm number

Details

returns the largest singular value of the matrix or vector

Value

Scalar Norm

Note

if k=1, absolute value; k=2 2-norm (rms); k>2, largest singular value.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
x = runif(10)  
  
Mnorm(x, k = 2)
```

nnz *Non-zeros*

Description

Number of non-zero elements in a vector

Usage

nnz(h)

Arguments

h vector

Value

integer

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
zip<-rnorm(15)
nnz(zip)
```

occam *Occam inversion*

Description

Occam's inversion

Usage

occam(afun, ajac, L, d, m0, delta)

Arguments

afun	character, function handle that computes the forward problem
ajac	character, function handle that computes the Jacobian of the forward problem
L	regularization matrix
d	data that should be fit
m0	guess at the model
delta	cutoff to use for the discrepancy principle portion

Value

vector, model found

Note

This is a simple brute force way to do the line search. Much more sophisticated methods are available. Note: we've restricted the line search to the range from 1.0e-20 to 1. This seems to work well in practice, but might need to be adjusted for a particular problem.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

bayes

phi *Integral of Normal Distribution*

Description

normal distribution and returns the value of the integral

Usage

phi(x)

Arguments

x endpoint of integration (scalar)

Value

value of integral

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

erf

Examples

```
x <- 1.0
##  pracma::erf(x)
phi(x)
phiinv( phi(x) )
```

phiinv

Inverse Normal Distribution Integral

Description

Calculates the inverse normal distribution from the value of the integral

Usage

```
phiinv(x)
```

Arguments

x endpoint value of integration (scalar)

Value

value of integral (scalar)

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

phi

Examples

```
x <- 1.0
## pracma::erf(x)
phi(x)
phiinv( phi(x) )
```

picard_vals

Picard plot

Description

Picard plot parameters for subsequent plotting.

Usage

```
picard_vals(U, sm, d)
```

Arguments

U	the U matrix from the SVD or GSVD
sm	singular values in decreasing order, or the GSVD lambdas divided by the mus in decreasing order
d	data to fit, right hand side

Details

The Picard plot is a method of helping to determine regularization schemes.

Value

List:

utd	the columns of U transposed times d
utd_norm	utd./sm

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

GSVD

Examples

```
####
n = 20
G = shawG(n,n)
spike = rep(0,n)
spike[10] = 1
dspiken = G

set.seed(2015)
dspiken = dspiken + 6e-6 *rnorm(length(dspiken))
Utube=svd(G);
U = Utube$u
V = Utube$v
S = Utube$d
s=Utube$d
R3 = picard_vals(U,s,dspiken);
utd = R3$utd
utd_norm= R3$utd_norm
#### Produce the Picard plot.

x_ind=1:length(s);
##
plot( range(x_ind) , range(c(s ,abs(utd),abs(utd_norm))),
      type='n', log='y', xlab="i", ylab="" )
lines(x_ind,s, col='black')
points(x_ind,abs(utd), pch=1, col='red')
points(x_ind,abs(utd_norm), pch=2, col='blue')

title("Picard Plot for Shaw Problem")
```

plotconst

Plot constant model

Description

Plots a model in piecewise constant form over n subintervals, where n is the length of x .

Usage

```
plotconst(x, l, r, ...)
```

Arguments

x	model to be plotted
l	left endpoint of plot

r right endpoint of plot
... graphical parameters

Details

Used for plotting vector models

Value

graphical side effects

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

linesconst

Examples

```
zip = runif(25)
plotconst(zip, 0, 1 )
linesconst(runif(25) , 0, 1 , col='red' )
```

quadlin

Lagrange multiplier technique

Description

Quadratic Linearization

Usage

```
quadlin(Q, A, b)
```

Arguments

Q positive definite symmetric matrix
A matrix with linearly independent rows
b data vector

Details

Solves the problem: $\min (1/2) t(x)*Q*x$ with $Ax = b$. using the Lagrange multiplier technique, where Q is assumed to be symmetric and positive definite and the rows of A are linearly independent.

Value

list:

x vector of solution values
 λ Lagrange multiplier

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
###%   Radius of the Earth (km)
      Re=6370.8;
      rad = 5000
      ri=rad/Re;

      q=c(1.083221147, 1.757951474)
      H = matrix(rep(0, 4), ncol=2, nrow=2)

      H[1,1]=1.508616069 - 3.520104161*ri + 2.112062496*ri^2;
      H[1,2]=3.173750352 - 7.140938293*ri + 4.080536168*ri^2;
      H[2,1]=H[1,2];
      H[2,2]=7.023621326 - 15.45196692*ri + 8.584426066*ri^2;
      A1 =quadlin(H,t(q), 1.0 );
```

 rnk

Rank of Matrix

Description

Return the rank of a matrix. Not to be confused with the R function rank.

Usage

```
rnk(G, tol = 1e-14)
```


Arguments

G Matrix
tol machine tolerance for small numbers

Details

Number of singular values greater than tol.

Value

integer, number of non-zero singular values

Note

duplicate the matlab function rank

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

svd

Examples

```
hilbert <- function(n) { i <- 1:n; 1 / outer(i - 1, i, "+") }  
X <- hilbert(9)[1:6]  
rnk(X)
```

setDesignG

Set a Design Matrix.

Description

Creata design matrix for simulating a tomographic inversion on a simple grid.

Usage

```
setDesignG()
```

Details

Set up a simple design matrix for tomographic in version. This is used in examples and illustrations of tomographics and matrix inversion methods.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
G = setDesignG()

### show the 56-th row
g = matrix( G[56,] , ncol=16, nrow=16)
imagesc(g)

## Not run:
### show total coverage
zim = matrix(0 , ncol=16, nrow=16)
for(i in 1:dim(G)[1])
{
g = matrix( G[i,] , ncol=16, nrow=16)
zim =zim + g
}
image(zim)

## End(Not run)
```

shawG

Shaw Model of Slit Diffraction

Description

Creates the design matrix for the Shaw inverse problem of diffraction through a narrow slot.

Usage

```
shawG(m, n)
```

Arguments

m	integer, number of rows
n	integer number of columns

Details

See Aster's book for a details explanation.

Value

Matrix used for creating data and inversion.

Author(s)

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References

C. B. Shaw, Jr., "Improvements of the resolution of an instrument by numerical solution of an integral equation", J. Math. Anal. Appl. 37: 83-112, 1972.

Examples

```
n = 20
G = shawG(n,n)

spike = rep(0,n)
spike[10] = 1

dspiken = G %% spike

plot(dspiken)
```

sirt

SIRT Algorithm for sparse matrix inversion

Description

Row action method for inversion of matrices, using SIRT algorithm.

Usage

```
sirt(A, b, tolx, maxiter)
```

Arguments

A	Design Matrix
b	vector, Right hand side
tolx	numeric, tolerance for stopping
maxiter	integer, Maximum iterations

Details

Iterates until conversion

Value

Solution vector

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Lees, J. M. and R. S. Crosson (1989): Tomographic inversion for three-dimensional velocity structure at Mount St. Helens using earthquake data, *J. Geophys. Res.*, 94(B5), 5716-5728.

See Also

art, kac

Examples

```
set.seed(2015)
G = setDesignG()
### Setup the true model.
mtruem=matrix(rep(0, 16*16), ncol=16,nrow=16);

mtruem[9,9]=1; mtruem[9,10]=1; mtruem[9,11]=1;
mtruem[10,9]=1; mtruem[10,11]=1;
mtruem[11,9]=1; mtruem[11,10]=1; mtruem[11,11]=1;
mtruem[2,3]=1; mtruem[2,4]=1;
mtruem[3,3]=1; mtruem[3,4]=1;

### reshape the true model to be a vector
mtruev=as.vector(mtruem);

### Compute the data.
dtrue=G %*% mtruev;

### Add the noise.

d=dtrue+0.01*rnorm(length(dtrue));

msirt<-sirt(G,d,0.01,200)
par(mfrow=c(1,2))
imagesc(matrix(mtruem,16,16) , asp=1 , main="True Model" );

imagesc(matrix(msirt,16,16) , asp=1 , main="SIRT Solution" );
```

Description

Inverse T-distribution, qt

Usage

```
tinv(p, nu)
```

Arguments

p	P-value
nu	degrees of freedom

Details

Wrapper for qt

Value

Quantile for T-distribution

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

qt

Examples

```
tinv(.4, 10)
```

USV

Singular Value Decomposition

Description

Singular Value Decomposition

Usage

```
USV(G)
```

Arguments

G	Matrix
---	--------

Details

returns matrices U, S, V according to matlab convention.

Value

list:

U	Matrix
S	Matrix, singular values
V	Matrix

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

See Also

svd

Examples

```
hilbert <- function(n) { i <- 1:n; 1 / outer(i - 1, i, "+") }  
X <- hilbert(9)[,1:6]  
  
h = USV(X)  
  
print( h$U )
```

Vnorm

Vector 2-Norm

Description

Vector 2-Norm.

Usage

Vnorm(X)

Arguments

X numeric vector

Value

Numeric scale norm

Note

This function is intended to duplicated the matlab function norm.

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

Examples

```
V = Vnorm(rnorm(10))
```

vspprofile

Vertical Seismic Profile In 1D

Description

Example vertical 1-dimensional seismic profile used for setting up examples for inverse theory.

Usage

```
vspprofile(M = 50, N = 50, maxdepth = 1000, deltobs = 20,  
noise = 2e-04, M1 = c(9000, -6, 0.001))
```

Arguments

M	integer, number of rows in in design matrix G, default=50
N	integer, number of columns in design matrix G, default=50
maxdepth	Maximum depth of model, default = 1000
deltobs	integer, sampling interval in depth, default=20
noise	gaussian noise multiplier, default=2e-04
M1	3-vector, linear model for velocity versus depth model

Details

Vertical seismic profile in 1D dimension used for setting up examples in PEIP. Given a simple velocity profile, defined by input parameter M1 create the travel times and design matrix used for solving an inverse problem. The velocity model is defined as depth versus velocity, and the function inverts that from the slowness. Any model could be used to replace this model. The default model here is taken from an inversion in the Aster book.

Value

list:

G	M by N design matrix
tee	true travel times from model
t2	travel times with noise added
depth	depth samples of model
vee	velocity at the depths indicated
M	input M
N	input N
maxdepth	input maxdepth
deltobs	input delta observation
noise	input noise
M1	True model used for depth versus velocity

Author(s)

Jonathan M. Lees<jonathan.lees@unc.edu>

References

Aster, R.C., C.H. Thurber, and B. Borchers, *Parameter Estimation and Inverse Problems*, Elsevier Academic Press, Amsterdam, 2005.

Examples

```
V = vspprofile()
### plot quadratic velocity profile
plot(V$vee, -V$depth, main="VSP: velocity increasing with depth")
dobs = seq(from=V$deltobs, to=V$maxdepth, by=V$deltobs)
### plotdepth versus time (not linear)
plot(dobs, V$t2)
abline(lm(V$t2 ~ dobs) )
```


Index

* misc

Ainv, 3
art, 4
bartl, 5
bayes, 6
blf2, 7
cgls, 9
chi, 10
chi2cdf, 11
chi2inv, 12
dcost, 13
error.bar, 13
flipGSVD, 15
gcv_function, 17
gcvval, 16
get_l_rough, 18
ginv, 19
GSVD, 20
idcost, 22
imagesc, 23
irls, 25
irlsl1reg, 26
kac, 28
l_curve_corner, 32
l_curve_tgsvd, 33
l_curve_tikh_gsvd, 35
l_curve_tikh_svd, 37
linesconst, 29
Imarq, 30
loadMAT, 31
mcmc, 38
Mnorm, 41
nnz, 42
occam, 42
phi, 43
phiinv, 44
picard_vals, 45
plotconst, 46
quadlin, 47

rnk, 48
setDesignG, 49
shawG, 50
sirt, 51
tinv, 52
USV, 53
Vnorm, 54
vspprofile, 55

* package

PEIP-package, 2

Ainv, 3
art, 4

bartl, 5
bayes, 6
blf2, 7

cgls, 9
chi, 10
chi2cdf, 11
chi2inv, 12
contoursc (imagesc), 23

dcost, 13

error.bar, 13

flipGSVD, 15

gcv_function, 17
gcvval, 16
get_l_rough, 18
ginv, 19
GSVD, 20

idcost, 22
imagesc, 23
interp2grid, 24
irls, 25
irlsl1reg, 26

kac, [28](#)

l_curve_corner, [32](#)

l_curve_tgsvd, [33](#)

l_curve_tikh_gsvd, [35](#)

l_curve_tikh_svd, [37](#)

linesconst, [29](#)

lmarq, [30](#)

loadMAT, [31](#)

mcmc, [38](#)

Mnorm, [41](#)

nnz, [42](#)

occam, [42](#)

PEIP (PEIP-package), [2](#)

PEIP-package, [2](#)

phi, [43](#)

phiinv, [44](#)

picard_vals, [45](#)

plotconst, [46](#)

quadlin, [47](#)

rnk, [48](#)

setDesignG, [49](#)

shawG, [50](#)

sirt, [51](#)

tinvs, [52](#)

USV, [53](#)

Vnorm, [54](#)

vspprofile, [55](#)